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CONFIDENCE INTERVALS FOR SYSTEM
RELIABILITY AND AVAILABILITY OF
MAINTAINED SYSTEMS USING
MONTE CARLO TECHNIQUES

THESIS

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THESIS

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Preface

With the steady increase in complexity of equipment, in the stringency of operating conditions and in the positive identification of system effectiveness requirements, it is becoming harder to satisfy the requirements; and more and more emphasis is placed on preventive maintenance together with the speedy repair of replicated units as a means of achievement. Test equipment is also becoming more and more complex. Maintainability engineering is, therefore, an attempt to achieve some repair time objectives by specifying a combination of design and human factors together with the maintenance philosophy consistent with the other engineering and cost constraints which exist.

The purpose of this study is to present the confidence intervals for system reliability and availability of maintained system using Mont Carlo techniques. The current literature indicates that Mont Carlo techniques offer the only practical method of analyzing large scale systems with general failure and repair systems. It is hoped that this kind of study will serve as a useful introduction to the various types of problems which are of current interest.

I would like to thank my thesis Advisor, Professor A. H. Moore, for his most valuable advice and guidance during this study. I would also like to thank Dr. Joseph Cain, my reader, for his help during the study. Also, I am

grateful to Mrs. Barbara Folkeson for her help in typing this thesis. I am also grateful to Mrs. Betty Harris and Mrs. Mary Browning of the Air Force Institute of Technology Library for their help in obtaining several references. Finally I wish to recognize the wonderful efforts of my wife, Gawhra, who encouraged me to strive, to search, to study, and, ultimately, to succeed.

Mohamed S. E. Hasaballa

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ABSTRACT

This thesis presents the results of an extensive literature search for finding the confidence intervals for system reliability and availability of maintained systems using Mont Carlo techniques. The characteristics of system reliability and maintainability analysis are discussed. The basic Markovian failure and repair models are developed. A summary of the point estimates of availability, reliability of maintained systems, and the exact analytical methods are presented. Finally, the Bootstrap (Double Mont Carlo technique) is used to obtain the confidence limits for the availability of the maintained systems.

CONFIDENCE INTERVALS FOR SYSTEM RELIABILITY
AND AVAILABILITY OF MAINTAINED SYSTEMS
USING MONT CARLO TECHNIQUES

I. Introduction

The reliability and maintainability disciplines, as they exist today, has evolved primarily during the past two decades. The impetus for these disciplines grew largely out of military and space programs and was related to several principal considerations:

(a) the relatively high failure rates of equipment during the 1940's and early 1950's;

(b) the resulting sharp increase in the cost of procurement and maintenance;

(c) the continual increase in total parts and functional complexity; and

(d) the recognized need to have an effective appproach to exclude or minimize the conditions that contributed to faulty equipment.

In the evolution of the reliability and maintainability disciplines, many mathematical methodologies were developed to account for and explain the observations (failures or repairs) generated by various equipments or systems under consideration. Since the observations were (and continue to be) often random in nature, e. g , stocastic rather than

deterministic, probabilistic concepts were applied to correctly analyze the particular system. Since the Mont Carlo techniques offer the best and valuable method of analyzing large scale systems with general failure and repair systems, these techniques are used to calculate the confidence intervals for system reliability and availability of maintained systems.

Reliability is a yardstick of the capability of an equipment to operate without failures when put into service. Reliability predicts mathematically the equipment's behavior under expected operating conditions. More specifically, reliability expresses in numbers the probability an equipment will operate without failure for a given length of time in an environment for which it was designed.

Maintainability is a more widely known term. Specifically, it is defined as the probability that a failed system can be made operable in a specified interval of downtime. Maintenance actions can be classified into two categories. First, there is off-schedule maintenance necessitated by system in-service failure or malfunction. Its purpose is to restore system operation as soon as possible by replacing, repairing, or adjusting the component or components which cause interruption of service. Second, there is a scheduled maintenance at regular intervals; its purpose is to keep the system in a condition consistent with its built-in levels of performance, reliability, and, where applicable, safety.

Availability is defined as the probability that a system is operating satisfactorily at any point in time. The problem is to calculate the confidence intervals for system reliability and availability of maintained systems using Mont Carlo techniques.

To achieve the study objectives, an extensive search was made of the available literature. The results of this search and the ensuing study are reported in the following sections: Section II includes a survey of techniques to study reliability, maintainability, and availability of maintained systems; Section III contains a summary of point estimates of availability and reliability of maintained systems; Section IV contains the confidence limits for availabilities of maintained systems - exact analytical methods; and finally, Section V contains the Mont Carlo comparisons confidence limits for availability and reliability. Appendix A contains a list of major reference sources - that is, documents which contained a majority of the references used in the study. Appendix B is a reference bibliography containing literature which is of a general nature, yet pertinent to the second objective of the thesis. This information has been included as an aid to the reader for further study. Appendix C is a supplemental bibliography containing two types of references: those applicable to the thesis but not selected for detailed analysis and those which appear to be applicable but were not readily obtainable.

II. Survey of Techniques to Study Reliability, Maintainability, and Availability of Maintained Systems

Reliability and maintainability engineering activities are performed for the purpose of determining and improving the reliability and maintainability of a particular system. These activities often involve the analysis of physical processes which have probabilistic transition events, in which case the state of the process at any time t is a random variable. Reliability and maintainability analyses, therefore, rely heavily upon the techniques of probability theory.

Reliability, Maintainability, and Availability Defined

Reliability is the probability that a system will perform its intended function satisfactorily for at least a given time period under specified operating conditions. The probability of failure as a function of time can be defined by

$$p(t \leq t) = F(t), \quad t \geq 0 \qquad (2\text{-}1)$$

where t is a random variable denoting the failure time. Then F(t) is the probability that the system will fail by time t. If we define the reliability as the probability of success, or the probability that the system will perform its intended function at a time t, we can write (Ref 43:9)

$$R(t) = 1 - F(t) = p(t > t) \qquad (2\text{-}2)$$

where R(t) is the reliability function. If the time to failure random variable t has a density function f(t), then

$$R(t) = 1 - F(t) = 1 - \int_0^t f(\tau)\partial\tau = \int_t^\infty f(\tau)\partial\tau \qquad (2\text{-}3)$$

Maintainability is a more widely known term. Specifically, maintainability is defined as the probability that a failed system can be made operable in a specified interval of downtime. Here the downtime includes the total time that the system is out of service. Downtime is a function of the failure detection time, repair time, administrative time, and the logistics time connected with the repair cycle. Theoretically, for a product there exists a maintainability function. The maintainability function describes probabilistically how long a system remains in a failed state. Mathematically

$$p(t \le t) = M(t) \qquad (2\text{-}4)$$

where t is the total downtime.

Availability is defined as the probability that a system is operating satisfactorily at any point in time and considers only operating time and downtime, i.e., excluding the idle time. Availability is a measure of the ratio of the operating time of the system to the operating time plus downtime. Thus it includes both reliability and maintainability.

The reliability and maintainability fields have evolved as distinct disciplines since 1950. Both rely heavily upon mathematical methodologies. Since they are concerned

with probabilistic concepts, many well-established mathematical concepts and techniques are applicable. One of these concepts is the theory of Markov processes. Many of the determinants of the reliability and maintainability of a system are random processes (sequence of states), any given state of which is dependent only upon the previous state in the process. Such processes are, by definition, Markov process; few references that present Markov process theroy include applications of this technique to reliability and maintainability. Thus it is the purpose of this chapter to provide the Markov models for reliability and maintainability analysis.

The Basic Markovian Failure Models

The basic Markovian failure model (Ref 30:18) consists of two states and a single transition event. The system is in state 0 if it is operating satisfactorily, and it is in state 1 if it has failed. The transition event from state 0 to state 1 is a system failure. Figure 1 shows the graph of the basic Markovian model. At any point in time, if the system is in state 0, it can either fail with probability p_{01} or not fail with probability p_{00}. System failure, state 1, is considered to be the end of the process. Therefore, p_{11} may either be ignored or set equal to 1.

The Markovian properties of the model result from the dependence of the one-step transition probabilities on only the immediately previous state. To amplify, the probability

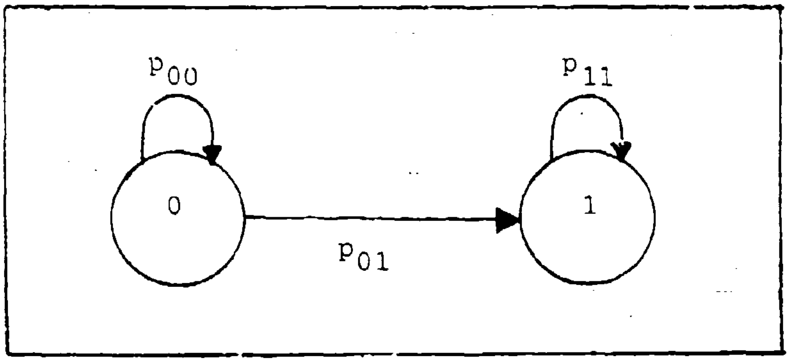


Fig. 1

Graph of the Basic Markovian Failure Model (from Ref 30:18)

of the occurrence or nonoccurrence of failure in the time interval, $(t+\Delta t)$; it is assumed to depend only upon the system being in the unfailed state at time t, regardless of the past history of the system. The one-step transition probabilities of the basic failure model depend upon and can be derived from the failure characteristics of the system when it is in state 0. The probability of a system failure in a small time t is

$$\int_{t}^{t+\Delta t} f(x)dx$$

where f(x) is the probability density of the time to failure of the system. Since failure cannot occur unless the system is operating, the conditional probability of failure in Δt, given the system has not failed prior to t, is

$$p_{01} = \frac{\int_t^{t+\Delta t} f(x)dx}{\int_t^{\infty} f(x)dx} \tag{2-5}$$

Equation (2-5) divided by Δt is the system failure rate (Ref 3:60).

The hazard function is defined as the limit of the failure rate as the interval approaches zero. Thus the hazard function is the instantaneous failure rate:

$$h(t) = \frac{f(t)}{R(t)} \tag{2-6}$$

also

$$f(t) = h(t) \exp\left[-\int_0^t h(\tau)\partial\tau\right] \tag{2-7}$$

Thus f(t), R(t), and h(t) are all related to each other and any one implies the other two (Ref 43:14).

Assuming stationary one-step transition probabilities in the basic failure model, $p_{01} = (q_{01}\cdot\Delta t)$, the failure rate of the system is $(q_{01}\cdot\Delta t)/\Delta t = q_{01}$, and $h(t) = q_{01}$. Letting $q_{01} = \lambda$, if the time to failure is described by an exponential density function, then

$$f(t) = \lambda e^{-\lambda t} \tag{2-8}$$

$$R(t) = \int_t^{\infty} \lambda e^{-\lambda\tau}\partial\tau$$

$$= \lambda \left. \frac{-e^{-\lambda\tau}}{\lambda} \right|_t^{\infty}$$

i.e.,

$$R(t) = e^{-\lambda t} \tag{2-9}$$

and

$$h(t) = \frac{f(t)}{R(t)} \tag{2-10}$$

Thus to achieve temporal homogeneity of the Markovian failure model, exponential failure times are usually assumed.

The basic Markovian model can be easily extended to include system states other than total operability and total failure. Figure 2 is the graph of such a model.

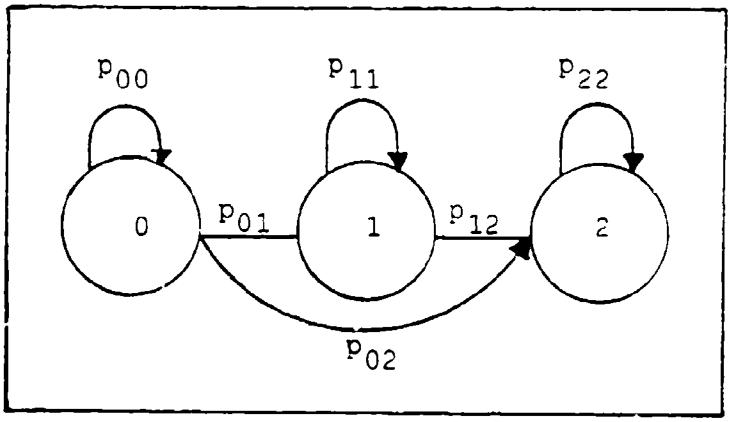


Fig. 2

Graph of a Three-State Failure Model (from Ref 30:20)

State 0 and 2 are "system operating" and "system failed," respectively. State 1 is any state of system degradation that changed the failure characteristics of the system but does not cause total system failure. The transition event from state 0 to state 1 is some form of system deterioration, and the transition events $0 \rightarrow 2$ and $1 \rightarrow 2$ are the two different modes of system failure that can occur. Because these three transition events differ physically, it is usually the

case that $p_{01} \neq p_{02} \neq p_{12}$. Extensions of these basic failure models to different system configurations require approximate addition and redefinition of the possible system states and their associated transition probabilities.

Application of the Basic Markovian Models to Reliability (Ref 30:21). System reliability, $R(t)$, is the probability that the system does not fail prior to time t. In terms of the basic Markovian failure model, Figure 1, $R(t)$ is the probability that the process does not progress to state 1 prior to time t for a continuous-time Markov failure model:

$$R(t) = 1 - p_1(t) = p_0(t) \qquad (2\text{-}11)$$

Assuming stationary transition probabilities,

$$p_0(t+\Delta t) = p_0(t) \cdot p_{00} = p_0(t)(1 - q_{01} \cdot \Delta t) \qquad (2\text{-}12)$$

$$p_0(t+\Delta t) = p_0(t) - p_0(t) q_{01} \cdot \Delta t$$

$$p_0(t+\Delta t) - p_0(t) = -p_0(t) q_{01} \cdot \Delta t$$

i.e.,

$$\frac{p_0(t+\Delta t) - p_0(t)}{\Delta t} = -p_0(t) q_{01}$$

Using the definition of the derivatives of a function,

$$p_0'(t) = -q_{01} \cdot p_0(t) \qquad (2\text{-}13)$$

Laplace transforming of equation (2-13)

$$sp_0(s) - p_0(t_0) = -q_{01} \cdot p_0(s)$$

Assuming $p_0(t_0) = 1$ and solving for $p_0(s)$

$$p_0(s) = \frac{1}{s + q_{01}} \qquad (2\text{-}14)$$

and

$$L^{-1}[p_0(s)] = p_0(t) = R(t) = e^{-q_{01} \cdot t} \qquad (2\text{-}15)$$

for the corresponding discrete-time failure model

$$R(K) = p_0(K) = p^0(p)^K \qquad (2\text{-}16)$$

assuming stationary p_{ij} and $p_0(0) = 1$

$$R(K) = (1 - q_{01} \cdot \Delta t)^K \qquad (2\text{-}17)$$

The expression for the reliability of the three-state system described by the failure model in Figure 2 differs from that of the two-state model in that the number of different paths resulting in system failure is increased. When the time parameter of the model is continuous,

$$R(t) = p_0(t) + p_1(t) = 1 - p_2(t) \qquad (2\text{-}18)$$

The probability $p_2(t)$ is determined as follows: from Figure 2 and the definition of $p_{ij}(\Delta t)$ (Ref 19)

$$||p_{ij}^{(\Delta t)}|| = \begin{bmatrix} q_{00}(\Delta t) & q_{01}(\Delta t) & q_{02}(\Delta t) \\ 0 & q_{11}(\Delta t) & q_{12}(\Delta t) \\ 0 & 0 & 1 \end{bmatrix}$$

Then

$$
\begin{aligned}
p_0(t+\Delta t) &= p_0(t)q_{00}\cdot\Delta t \\
&= p_0(t)[1 - (q_{01}\cdot\Delta t + q_{02}\cdot\Delta t)] \\
p_1(t+\Delta t) &= p_0(t)q_{01}\cdot\Delta t + p_1(t)q_{11}\cdot\Delta t \\
&= p_0(t)q_{01}\cdot\Delta t + p_1(t)(1 - q_{12}\cdot\Delta t) \\
p_2(t+\Delta t) &= p_0(t)q_{02}\cdot\Delta t + p_1(t)q_{12}\cdot\Delta t + p_2(t)
\end{aligned}
\tag{2-19}
$$

Taking derivatives,

$$
\begin{aligned}
p_0'(t) &= -(q_{01} + q_{02})p_0(t) \\
p_1'(t) &= q_{01}p_0(t) - q_{12}p_1(t) \\
p_2'(t) &= q_{02}p_0(t) + q_{12}p_1(t)
\end{aligned}
\tag{2-20}
$$

Laplace transforming equation (2-20) and assuming $p_0(t_0) = 1$ and $p_1(t_0) = p_2(t_0) = 0$, then

$$
\begin{aligned}
(s + q_{01} + q_{02})p_0(s) &= 1 \\
q_{01}p_0(s) - (s + q_{12})p_1(s) &= 0 \\
q_{02}p_0(s) + q_{12}p_1(s) - sp_2(s) &= 0
\end{aligned}
\tag{2-21}
$$

By Cramer's Rule,

$$
p_2(s) = \frac{\begin{vmatrix} s + q_{01} + q_{02} & 0 & 1 \\ q_{01} & -(s + q_{12}) & 0 \\ q_{02} & q_{12} & 0 \end{vmatrix}}{\begin{vmatrix} s + q_{01} + q_{02} & 0 & 0 \\ q_{01} & -(s + q_{12}) & 0 \\ q_{02} & q_{12} & -s \end{vmatrix}}
$$

$$p_2(s) = \frac{q_{01}q_{12} + q_{02}(s + q_{12})}{s(s + q_{12})(s + q_{01} + q_{02})} \qquad (2\text{-}22)$$

By partial fractions

$$p_2(s) = \frac{A}{s} + \frac{B}{s + q_{12}} + \frac{C}{s + q_{01} + q_{02}}$$

$$A = \lim_{s \to 0} \frac{q_{01}q_{12} + q_{02}(s + q_{12})}{(s + q_{12})(s + q_{01} + q_{02})}$$

$$= \frac{q_{01}q_{12} + q_{02}q_{12}}{q_{01}q_{12} + q_{02}q_{12}} = 1$$

$$B = \lim_{s \to -q_{12}} \frac{q_{01}q_{12} + q_{02}(s + q_{12})}{s(s + q_{01} + q_{02})}$$

$$= \frac{q_{01}q_{12}}{-q_{12}(q_{01} + q_{02} - q_{12})}$$

$$= - \frac{q_{01}}{(q_{01} + q_{02} - q_{12})}$$

To obtain the value of C we have

$$\frac{q_{01}q_{12} + q_{02}(s + q_{12})}{s(s + q_{01} + q_{02})(s + q_{12})} = \frac{A}{s} + \frac{B}{s + q_{12}} + \frac{C}{s + q_{01} + q_{02}}$$

i.e.,

$$\begin{aligned} q_{01}q_{12} + sq_{02} + q_{02}q_{12} &= A(s + q_{12})(s + q_{01} + q_{02}) \\ &\quad + Bs(s + q_{01} + q_{02}) \\ &\quad + Cs(s + q_{12}) \\ &= A(s^2 + sq_{01} + sq_{02} + sq_{12} \\ &\quad + q_{01}q_{12} + q_{02}q_{12}) + Bs^2 \\ &\quad + Bsq_{01} + Bsq_{02} + Cs^2 + Csq_{12} \end{aligned}$$

Taking the coefficients of s^2 in both sides

$$0 = As^2 + Bs^2 + Cs^2$$

$$\text{i.e.,}\quad A + B + C = 0$$

$$\text{so}\quad C = -(A + B)$$

$$C = -\left[1 - \frac{q_{01}}{q_{01} + q_{02} - q_{12}}\right]$$

$$= -\left[\frac{q_{12} - q_{02}}{q_{01} + q_{02} - q_{12}}\right]$$

$$= -\frac{q_{12} - q_{02}}{q_{12} - (q_{01} + q_{02})}$$

i.e.,

$$A = 1, \quad B = -\frac{q_{01}}{(q_{01} + q_{02} - q_{12})}, \quad C = \frac{-q_{12} - q_{02}}{q_{12} - (q_{01} + q_{02})} \qquad (2\text{-}23)$$

$$p_2(s) = \frac{1}{s} + \frac{(-q_{01}/q_{01} + q_{02} - q_{12})}{s + q_{12}}$$

$$+ \frac{(-(q_{12} + q_{02})/q_{12} - (q_{01} + q_{02})}{s + q_{01} + q_{02}}$$

$$L^{-1}[p_2(s)] = p_2(t) = 1 - B \exp - q_{12}t - C \exp - (q_{01} + q_{02})t$$

$$R(t) = 1 - p_2(t)$$

$$R(t) = B \exp - q_{12}t + C \exp - (q_{01} + q_{02})t \qquad (2\text{-}24)$$

The reliability expression for the discrete-time model of Figure 2 is

$$R(K) = p_0(K) + p_1(K) = 1 - p_2(K)$$

where $p_2(K)$ is the sum of all possible paths that will cause the system to be in state 2 after the K-th step of the process

$$p_2(K) = p_2(o) + p_1(0)p_{12}(K) + p_0(0)p_{02}(K) \qquad (2\text{-}25)$$

using the transition matrix method (Ref 19)

$$p_2(K) = (p^0 p^K)_3 \qquad (2\text{-}26)$$

i.e., the third (last) element in the vector resulting from the multiplication of the initial-state probabilities vector and the K-th step transition probability matrix. The reliability expressions for more complex Markovian failure models must be derived from the individaul model in terms of the possible states and the failure characteristics of the system being modeled. The general approacn, however, is the same as shown above.

The Basic Markovian Repair Model

The basic Markovian repair model (Ref 30:27) is shown in Figure 3. The system states of this model are identical to the states of the basic Markovian failure model: state 0 if the system is operating and state 1 if the system has failed. The transition event of this model is any repair that places a failed system in satisfactory operating condition. From the failed state 1, the system can either be repaired in t with probability p_{10} or not repaired in Δt with probability p_{11}. The repair process is completed when the system

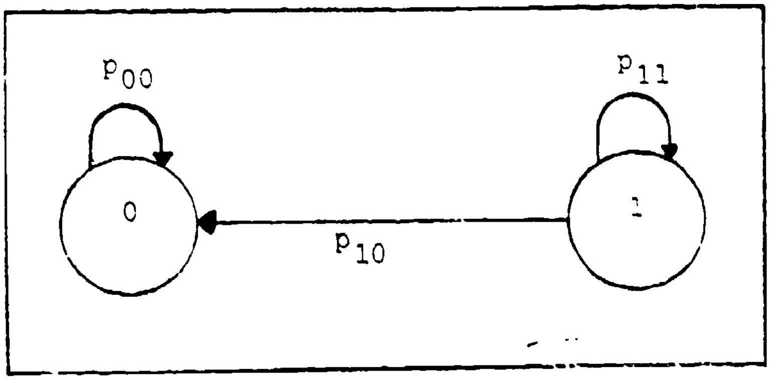


Fig. 3

Graph of the Basic Markovian Repair Model (from Ref 30:27)

is returned to operable condition. Thus, p_{00} = 1 the random variable, time-to-repair of a particular system is a function of the repair characteristics (i.e., accessibility, complexity, configuration, etc.) of the system and the characteristics of the repair activity (i.e., maintenance procedures, man power, technical proficiency, etc.). The transition probability, $p_{10}(t)$, is derived from the probability density, $g(t)$, of the system time-to-repair. When the repair-time characteristics of the system remain unchanged regardless of the past history of the system, the density, $g(t)$, remains unchanged and the repair model is Markovian. The limit, as $t \to 0$, of the conditional probability that the system will be repaired in $(t+\Delta t)$ given that it is failed at t is the instantaneous repair rate, analogous to the hazard rate of the basic Markovian failure model. To achieve stationary p_{10}, exponential repair times are often assumed, in which case, $p_{10} = q_{10} \cdot \Delta t$, where q_{10} is a constant repair rate.

Because repair is the reverse process of failure, except for the nature and direction of the transition events as discussed above, the operation of the basic Markovian repair model is similar to that of the basic Markovian model.

Application of the Basic Markovian Repair Model to Maintainability (Ref 30:28). System maintainability, $M(t)$, is the probability that a failed sys- is repaired within a specified time. In terms of the basic Markovian repair model, Figure 3, $M(t)$ is the probability that the system transition from state 1 to state 0 within the time interval (t_0-t). For the continuous-time Markovian repair model

$$M(t) = p_0(t) = 1 - p_1(t) \qquad (2\text{-}27)$$

Assuming p_{10} stationary and $p_1(t_0) = 1$

$$M(t) = 1 - e^{-q_{10}t} \qquad (2\text{-}28)$$

The corresponding expression for $M(t)$ of the basic discrete-time repair model is

$$M(K) = 1 - p_1(K) = 1 - (1 - q_{10}\cdot t)^K \qquad (2\text{-}29)$$

As it is used above, and consistent with the definition of maintainability, repair is corrective maintenance. Each different mode of failure of the system requires a different repair action. Thus the basic Markovian repair model can be extended to describe the system at any desired level of detail. Figure 4 shows an n-th extension of the basic

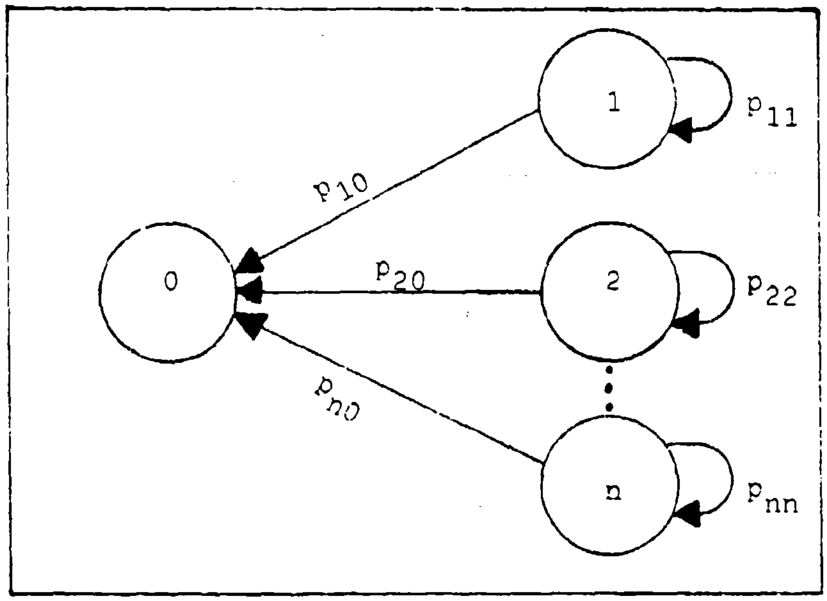


Fig. 4

n-th Extension of the Basic Markovian Repair Model

(from Ref 30:30)

Markovian repair model. State 0 is the "system operating satisfactorily" and states 1 through n represent system failure due to the possible modes or malfunctions by which the system can fail. It is reasonable to assume that the time-to-repair for each different mode of failure is different and that $p_{10} = p_{20} = \ldots = p_{n0}$. For this model,

$$M(t) = \sum_{i=1}^{n} p_i(t_0) \cdot p_{i0}(t) \qquad (2\text{-}30)$$

Other extensions of the basic repair model depend upon system configuration and system repair characteristics.

Specific Applications of Markov Processes to Reliability and Maintainability

Numerous extensions and combinations of the basic Markovian failure and repair models were found during the literature search portion of this study. Representative models showing the applicability of Markov process techniques to the estimation of system reliability and maintainability are presented in this section. Specific attention is given to Markov process applications in redundant systems and maintenance systems. Examples of the combined failure and repair models used to determine the system availability are also discussed.

Redundant Systems. Shooman (Ref 78) presents an extension of the basic Markovian failure model: a two element non-repairable system. Figure 5 shows the graph of the system. The author makes provision in the model for failure hazards which are a function of time, i.e., non-homogeneous. This discussion, however, considers only the homogeneous model with constant failure hazards, $p_{jK} = \lambda_i$.

The states of the system are defined as follows:

state 0 = both elements operating;

state 1 = element 1 failed, element 2 operating;

state 2 = element 2 failed, element 1 operating;

state 3 = both elements failed;

and the transitions may be described as follows:

λ_i = the failure rate associated with the two states in question

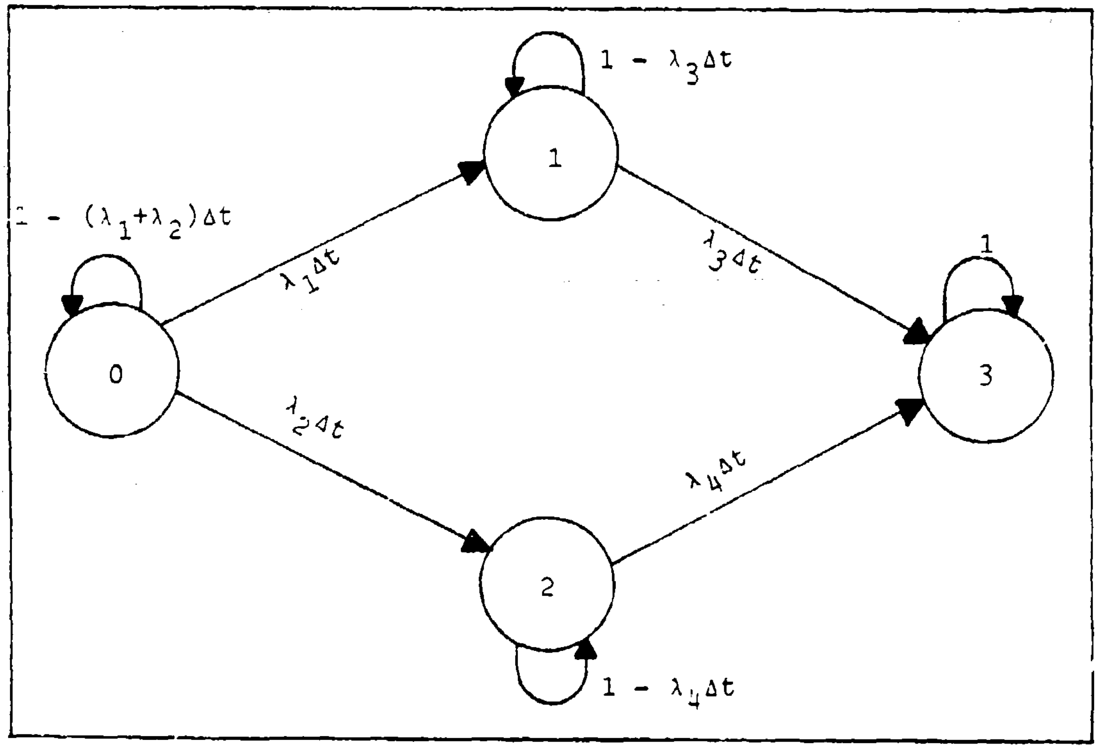


Fig. 5

Graph of Two-Element Non-repairable System

$\lambda_1 \Delta t$ = the transition probability due to element failure in time interval Δt.

Using difference-differential equations procedures, the author develops the following state probability expressions:

$$p_0(t+\Delta t) = [1 - (\lambda_1+\lambda_2)\Delta t]p_0(t) \qquad (2\text{-}31)$$

$$p_1(t+\Delta t) = [\lambda_1 \Delta t]p_0(t) + [1 - \lambda_3 \Delta t]p_1(t) \qquad (2\text{-}32)$$

$$p_2(t+\Delta t) = [\lambda_2 \Delta t]p_0(t) + [1 - \lambda_4 \Delta t]p_2(t) \qquad (2\text{-}33)$$

$$p_3(t+\Delta t) = [\lambda_3 \Delta t]p_1(t) + [\lambda_4 \Delta t]p_2(t) + 1p_3(t) \qquad (2\text{-}34)$$

The solutions to these equations, with initial conditions $p_0(0) = 1$ and $p_1(0) = p_2(0) = p_3(0) = 0$, are

$$p_0(t) = e^{-(\lambda_1 + \lambda_2)t} \tag{2-35}$$

$$p_1(t) = \frac{\lambda_1}{\lambda_1 + \lambda_2 - \lambda_3}\left[e^{-\lambda_3 t} - e^{-(\lambda_1 + \lambda_2)t}\right] \tag{2-36}$$

$$p_2(t) = \frac{\lambda_2}{\lambda_1 + \lambda_2 - \lambda_4}\left[e^{-\lambda_4 t} - e^{-(\lambda_1 + \lambda_2)t}\right] \tag{2-37}$$

$$p_3(t) = 1 - [p_0(t) + p_1(t) + p_2(t)] \tag{2-38}$$

For a two-element redundant (parallel) system, one failure can be tolerated; and hence, there are three successful states: $p_0(t)$, $p_1(t)$, and $p_2(t)$. Since these states are mutually exclusive, the expression for system reliability is

$$R(t) = p_0(t) + p_1(t) + p_2(t)$$

or

$$R(t) = e^{-(\lambda_1 + \lambda_2)t} + \frac{\lambda_1}{\lambda_1 + \lambda_2 - \lambda_3}\left[e^{-\lambda_3 t} - e^{-(\lambda_1 + \lambda_2)t}\right] + \frac{\lambda_2}{\lambda_1 + \lambda_2 - \lambda_4}\left[e^{-\lambda_4 t} - e^{-(\lambda_1 + \lambda_2)t}\right] \tag{2-39}$$

The author points out the complete generality of equation (2-39) and its usefullness in determining the reliability of any two-element redundant system with constant hazard elements (Ref 78:235). For example, if the hazard functions are the same regardless of the state of the system, i.e.,

$\lambda_1 = \lambda_4$ and $\lambda_2 = \lambda_3$, then

$$R(t) = e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2)t} \qquad (2\text{-}40)$$

If the elements are identical and the failure rate with both operating is λ_b and with a single element operating λ_s, then $\lambda_1 = \lambda_2 = \lambda_b$ and $\lambda_3 = \lambda_4 = \lambda_s$ which yields

$$R(t) = \frac{2\lambda_b e^{-\lambda_s t} - \lambda_s e^{-2\lambda_b t}}{2\lambda_b - \lambda_s} \qquad (2\text{-}41)$$

Finally, if $\lambda_s = \lambda_b = \lambda$, then

$$R(t) = e^{-\lambda t}(2 - e^{-\lambda t}) \qquad (2\text{-}42)$$

(Ref 43:217) shows that the expected time to system failure, found by integrating equation (2-40) over the range of t, is

$$E_s(t) = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} - \frac{1}{\lambda_1+\lambda_2} \qquad (2\text{-}43)$$

when all the units have the same failure rate λ. Then equation (2-43) gives the mean time to failure for a two component system as

$$E_s(t) = \frac{2}{\lambda} - \frac{1}{2\lambda} = 3/2\lambda$$

for a three component system, we have

$$E_s(t) = 11/6\lambda$$

and in general for n component system, the mean time to failure is

$$E_s(t) = \sum_{i=1}^{n} \theta / i \qquad (2\text{-}44)$$

where $\theta = \frac{1}{\lambda}$. Here it can be seen that the marginal gain in the mean time to system failure decreases with each subsystem added.

If equation (2-39) represents a two-element standby system initially in state zero, then $\lambda_1 = \lambda_A$ and $\lambda_2 = 0$ since element 2 cannot fail prior to failure of element 1 because it is an unenergized state. Thus $\lambda_3 = \lambda_B$ and λ_4 need not be specified since if $p_0(0) = 1$ and $\lambda_2 = 0$, state 2 has a probability which is zero. With these substitutions, equation (2-39) becomes

$$R(t) = \frac{\lambda_A e^{-\lambda_B t}}{\lambda_A - \lambda_B} - \frac{\lambda_B e^{-\lambda_A t}}{\lambda_A - \lambda_B} \qquad (2\text{-}45)$$

In a similar application, Zorger (Ref 91) computes the reliability of a "maintained" or repairable two-unit system. In this model, the author assumes the failure rates are the same regardless of the state of the system, e.g., $\lambda_1 = \lambda_4$ and $\lambda_2 = \lambda_3$. The repair rate of component i, μ_i, multiplied by the time interval Δt gives the probability of transition due to a repair. The graph of this system is shown in Figure 6.

Due to the fact that Δt is considered infinitesimal, the probability of a double transition in Δt is understood to be zero. System reliability is computed on the basis of the first time state 3 is reached; subsequent system repair has no effect on the reliability of the system. Hence,

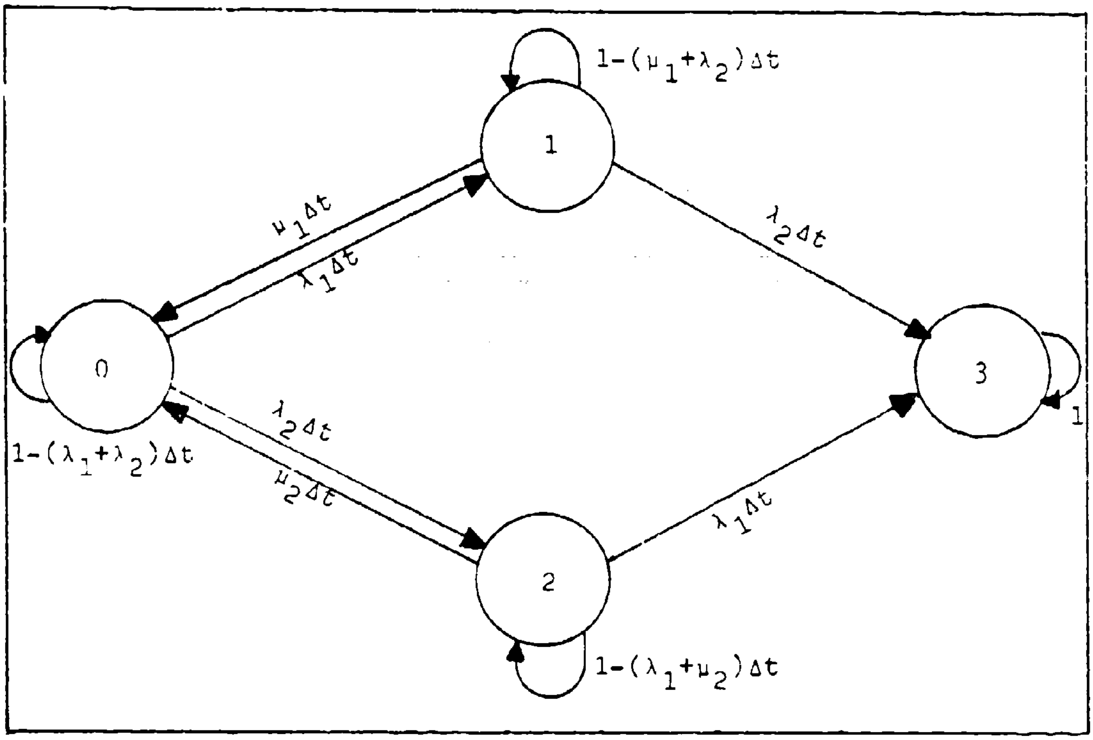


Fig. 6

Graph of Two-Element Repairable System

$p_{32}(t)$ and $p_{31}(t)$ are also considered to be zero. The author shows that the reliability of the repairable system is

$$R(t) = 1 - L^{-1}p_0(s) \qquad (2\text{-}46)$$

where

$$p_0(s) = \Delta sp_0(s)/\Delta(s)$$

$$\begin{aligned}\Delta sp_0(s) = {} & \lambda_1\lambda_2 A_{33}[2s + \lambda_1 + \lambda_2 + \mu_1 + \mu_2] \\ & + \lambda_2 A_{22}[s^2 + s(2\lambda_1 + \lambda_2 + \mu_2) \\ & + \lambda_1^2 + \lambda_1\lambda_2 + \mu_2\lambda_1 + \mu_2\lambda_2]\end{aligned}$$

$$+ \lambda_1 A_{11}[s^2 + s(\lambda_1 + 2\lambda_2 + \mu_1) + \lambda_2^2$$
$$+ \lambda_1\lambda_2 + \mu_1\lambda_2 + \mu_1\lambda_1] + A_{00}[s^3 +$$
$$+ s^2(2\lambda_1 + 2\lambda_2 + \mu_1 + \mu_2) + s(3\lambda_1\lambda_2$$
$$+ \mu_2\lambda_2 + \mu_1\lambda_1 + \lambda_1^2 + \lambda_2^2 + \mu_2\lambda_1 + \mu_1\lambda_2$$
$$+ \mu_1\mu_2) + \lambda_1^2\lambda_2 + \lambda_1\lambda_2^2 + \mu_2\lambda_1\lambda_2$$
$$+ \mu_1\lambda_1\lambda_2 \qquad (2\text{-}47)$$

With A_{11} representing the initial condition $p_1(t_0)$ and

$$\Delta s = -s\left\{(s + \lambda_1 + \mu_2)(s + \lambda_2 + \mu_1)(s + \lambda_1 + \lambda_2) - \mu_2\lambda_2(s + \lambda_2 + \mu_1) - \mu_1\lambda_1(s + \lambda_1 + \mu_2)\right\} \qquad (2\text{-}48)$$

letting $A_{00} = 1$, $A_{11} = 1$, $A_{22} = 0$, $\mu_1 = \mu_2 = \mu$ and $\lambda_1 = \lambda_2 = \lambda$, the above expressions are greatly simplified

$$R(t) = \frac{Ae^{Bt} - Be^{At}}{A - B} \qquad (2\text{-}49)$$

where

$$A,\ B = \frac{(-3\lambda + \mu) \pm \sqrt{\lambda^2 + 6\mu\lambda + \mu^2}}{2} \qquad (2\text{-}50)$$

using equation (2-49), the author shows that the maintained system MTBF is $(3\lambda + \mu)/(2\lambda^2)$.

Zorger makes no mention of system maintainability. It can, however, be derived from the repairable system model by considering only the repair characteristics of the system. Assuming one repairman is available, the system is

repaired when either element 1 or element 2 is operating. Also, $p_{30} = 0$ due to the infinitesimal probability of two repairs in Δt. Hence,

$$M(t) = p_2(t) + p_1(t) = 1 - p_3(t) \qquad (2\text{-}51)$$

By the methods used above,

$$p_3(t+\Delta t) = p_3(t)(1 - \mu_1\Delta t)(1 - \mu_2\Delta t)$$

$$= p_3(t) - (\mu_1\Delta t + \mu_2\Delta t)p_3(t) \qquad (2\text{-}52)$$

$$p_3'(t) = -(\mu_1 + \mu_2)p_3(t) \qquad (2\text{-}53)$$

$$sp_3(s) - A_{33} = -(\mu_1 + \mu_2)p_3(s) \qquad (2\text{-}54)$$

From the definition of maintainability, $A_{33} = 1$ and $A_{22} = A_{11} = 0$. Thus

$$sp_3(s) - 1 = -(\mu_1 + \mu_2)p_3(s)$$

i.e.,

$$p_3(s)(s + \mu_1 + \mu_2) = 1$$

$$p_3(s) = \frac{1}{s + \mu_1 + \mu_2} \qquad (2\text{-}55)$$

$$p_3(t) = e^{-(\mu_1 + \mu_2)t} \qquad (2\text{-}56)$$

$$M(t) = 1 - e^{-(\mu_1 + \mu_2)t} \qquad (2\text{-}57)$$

Messinger (Ref 56) considers a "K-out-of-h" redundant structure composed of identical elements with constant

failure rates and no repair capability. The system will operate when at least any K of its components are operating. The states of the system are defined to be the number of failed components; thus, the number of system states, n + 1, increases linearly with the number of components, n, in the structure. The component failure rate in the i-th state (i.e., when i failures have occurred) is denoted by λ_i. The graph or transition diagram for this system is shown in Figure 7. The reliability of the K-out-of-n structure is computed by summing the probabilities of successful system states 0,1,2,...,n-K , i.e.,

$$R(t) = \sum_{i=0}^{n-K} p_i(t) \qquad (2\text{-}58)$$

The author points out that considerable computational effort can be saved (especially if K is near n) by grouping all of the system failed states into a single collective failed state. (This is essentially equivalent to partitioning the transition matrix into successful and unsuccessful states as considered by Sandler (Ref 73:100).) This grouping is permissible because the individual probabilities of being in the failed states are of no interest in calculating system reliability. Once the system enters a failed state, it cannot, without repair, return to a good state. Thus, the system can be modeled by n - K + 1 good states and one collective failed state, a total of n - K + 2 states. Using a collective failed state, the transition diagram would

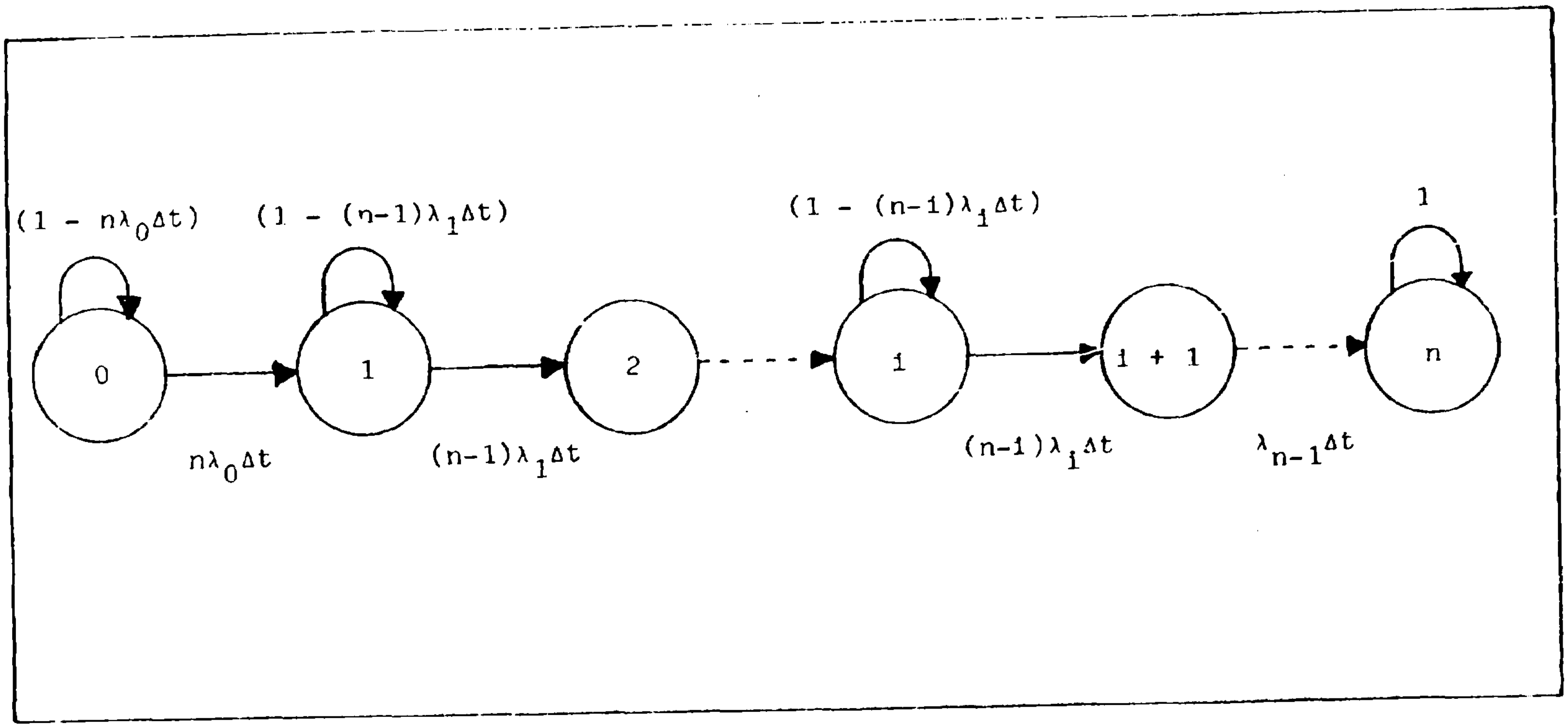


Fig. 7

Transition Diagram for K-out-of-n System (from Ref 58:145)

appear as in Figure 7 with state (n-1) relabeled as (n-K) and state (n) relabeled as (F) for the failed state. The transition probabilities would also require appropriate modification.

In the special case where the hazard rate is assumed proportional to stress, i.e.,

$$\lambda_i = \frac{n}{n - i} \lambda_0 \quad \text{for } i = 1,2,\ldots,n-1 \tag{2-59}$$

and the components are all assumed initially operational, the author shows that the system reliability is

$$R(t) = e^{-n\lambda t} \sum_{i=0}^{n-K} \frac{(n\lambda t)^i}{i!} \tag{2-60}$$

Reliability production formulas for standby redundant systems have been developed by numerous authors. Equation (2-45) is an example of the reliability expression for a two-element standby system. In general, the reliability of a structure composed of N identical elements, of which N - 1 are in stanby (assuming exponential distributed failure times and off-line failure rate equal to zero) is given by

$$R(t) = e^{-\lambda t} \sum_{J=0}^{N} \frac{(\lambda t)^J}{J!} \tag{2-61}$$

Benning (Ref 9) employs Markovian techniques to derive the reliability expression for a structure involving

N identical components of which at least K ($K < N$) are required for system operation while $N - K$ are in standby. The author shows that

$$R(t) = e^{-K\lambda t} \sum_{j=0}^{N-K} \frac{(K\lambda t)^j}{j!} \qquad (2\text{-}62)$$

and that the mean life (ML) of this configuration is

$$ML = \frac{N - K + 1}{K\lambda} \qquad (2\text{-}63)$$

For a similar K-out-of-N system in which all N components are on-line, the mean life is

$$ML = \frac{1}{\lambda} \sum_{i=0}^{N-K} \frac{1}{(N - i)} \qquad (2\text{-}64)$$

Comparing equation (2-63) and equation (2-64) term-by-term shows that every term in equation (2-64) is less than or equal to each term in equation (2-63). This result, as the author explains, shows that the K-out-of-N standby structure is more reliable than the on-line structure providing switching is perfectly reliable.

Kapur (Ref 43:221) shows that for the perfect switching case $R_s^n(t)$ - the reliability function for a standby system that has n subsystems - is

$$R_s^n(t) = e^{-\lambda t} \sum_{i=0}^{n-1} (\lambda t)^i / i! \qquad (2\text{-}65)$$

where λ is the failure rate and is identical for each standby

unit. However, for larger k, the design of switching circuitary would be quite complex if it were done automatically. Hence, it appears that standby redundancy would be most useful in those applications in which manual switching or replacement of components would be tolerated (Ref 13:137).

Epstein and Weinstock (Ref 28) consider a similar system composed of N units of which m are on-line and N-m are off-line (standby). Of the m on-line units, at least K must be operating properly except for an "acceptable downtime" which is less than or equal to t_0 units of time. In other words, even if less than K units are operational, the system is still considered to be performing satisfactorily as long as this condition persists for less than time t_0. It is assumed that each element of the system has independent exponential failure and repair times and that there are N available repairmen. The authors consider the system to be in either one of two states, the good state or the bad state. If the system has had at least one failure (less than K units operational) for longer than t_0 prior to the time in question, it is considered to be in a bad state. "Conditional availability" is defined by the authors as the probability of being in a particular state at time t conditioned on the fact that the system is not in a bad state at time t.

Strinivason (Ref 80) applies a Markov model to a standby redundant system where the switchover is not instantaneous. Switchover time is considered to be a random variable and is accumulated from the instant action is initiated to bring

the standby system to the active state, to the instant at which the standby unit becomes operational. The author assumes exponentially distributed failure times, an arbitrary repair distribution, and an arbitrary distribution for the probability that switchover is completed in (0, t). After deriving a lengthy equation for the probability distribution of first time to failure, the author then obtains an explicit expression for the expected time to system failure. His results show that if cost factors are of no consideration and if the objective is to have maximum MTBF, "The best policy is to initiate switching action . . ." just at the moment the subsystem becomes standby (Ref 80:176).

Kapur (Ref 43:221) considers the case of imperfect switching. He first looked at a situation where the switch simply fails to operate when called upon. The probability that the switch performs when required is p_s. For the two-unit standby system, it was shown that

$$R_s^2(t)' = R_1(t) + p_s \int_0^t f_1(t_1) R_2(t - t_1) \partial t_1 \qquad (2\text{-}66)$$

and for a three-unit system

$$R_s^3(t)' = R_s^2(t)' + p_s^2 p_3 \qquad (2\text{-}67)$$

He considered that the switch is a complex piece of equipment and has a constant failure rate of λ_s. Thus the reliability function for the switch is

$$R_s(t) = e^{-\lambda_s t}, \quad t \geq 0$$

and the switch can fail before it is needed. When he considers the two-unit standby system, the reliability at time t is

$$R_s^2(t)'' = p[(t_1 > t) \cup (t_1 \leq t \cap t_s > t_1 \cap t_2 > t - t_1)] \quad (2\text{-}68)$$

where t_s is the random variable representing time to switch failure. Figure 8 shows the success models for a two-unit standby system.

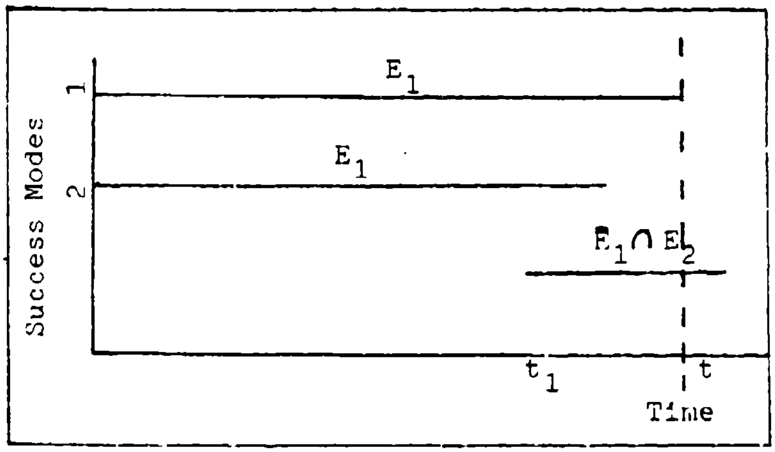


Fig. 8

Success Modes for a Two-Unit Standby System (Ref 43:219)

Equation (2-68) becomes

$$R_s^2(t)'' = R_1(t) + \int_0^t f_1(t_1)R_s(t_1)R_2(t - t_1)\partial t \quad (2\text{-}69)$$

by substitution for $R_s(t_1)$

$$R_s^2(t)'' = R_1(t) + \int_0^t f_1(t_1)e^{-\lambda_s t_1}R_2(t - t_1)\partial t_1 \quad (2\text{-}70)$$

The author considers the special case where all subsystems have a constant failure rate λ, then equation (2-70) reduces to

$$R_s^2(t)'' = e^{-\lambda t}\left[1 + \frac{\lambda}{\lambda_s}(1 - e^{-\lambda_s t})\right], \quad t \geq 0 \quad (2\text{-}71)$$

He finally considers the case of three-unit standby systems which have a constant failure rate λ; he deduced

$$R_s^3(t)'' = e^{-\lambda t}\left[1 + \frac{\lambda}{\lambda_s}(1 - e^{-\lambda_s t})\right] + e^{-\lambda t}(\lambda/\lambda_s)^2\left[1 - e^{-\lambda_s t} - \lambda_s t e^{-\lambda_s t}\right], \quad t \geq 0 \quad (2\text{-}72)$$

Anderson (Ref 2) employs a Markov chain model to analyze the reliability of a special class of redundant systems. These systems include those which operate in a standby mode for a long period of time in anticipation of participation in a single mission. The system consists of N modules of which q are active on-line spares. The system is in operating condition when at least N - q modules are operating; the system has failed when more than q modules have failed. The following assumptions are made:

(1) The module rate (λ) is constant.

(2) The module repair rate (μ) is also constant and independent of the number of failed modules (one module is repaired at a time).

(3) Repairs may be made during the standby interval but not during the mission interval.

(4) The system has been in the standby mode sufficiently long to insure its being in a steady-state condition, with respect to reliability, when it enters the mission interval.

The failure state diagram for this system is shown in Figure 9. The state number corresponds to the number of failed modules. Transitions from state k to k + 1 occur at failure rate λ_K and transition from state k to k - 1 occur at repair rate μ_K.

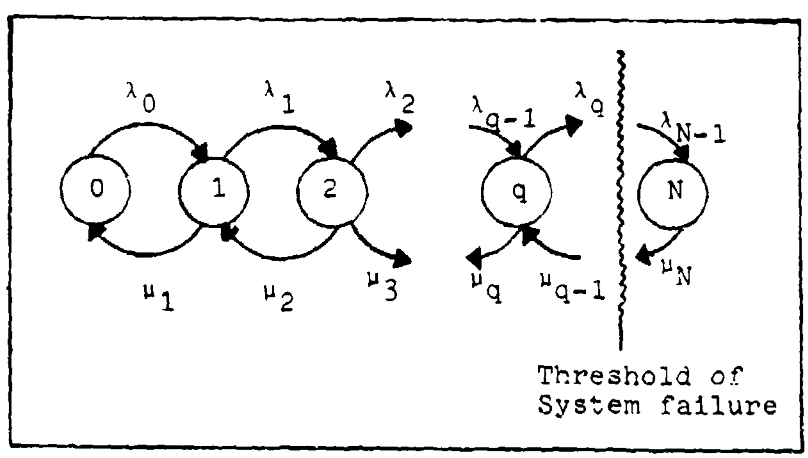


Fig. 9

Failure State Diagram (from Ref 2:22)

The general differential equation for this system is the same as those developed by McGregor (Ref 57). Anderson manipulates these general equations to determine the probability of mission success, p_{ms}, and the expected downtime per year, DT/Y_r, for the assumptions and constraints (notably the initial conditions) of the given system. Since there is no repair during the mission interval, the probability of mission success is simply the sum of the individual probabilities of being in an unfailed state, i.e.,

$$p_{ms} = \sum_{K=0}^{q} p_K(t_m) \qquad (2\text{-}73)$$

The solution for $p_K(t_m)$ involves the solution of the set of simultaneous differential equations which apply during the mission interval. The author derives an explicit equation for $p_K(t_m)$ which is expressed in matrix form as

$$\begin{bmatrix} p_0(t_m) \\ p_1(t_m) \\ p_2(t_m) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ p_n(t_m) \end{bmatrix} = \begin{bmatrix} \phi_{00} & 0 & 0 & \cdots & 0 \\ \phi_{10} & \phi_{11} & 0 & \cdots & 0 \\ \phi_{20} & \phi_{21} & \phi_{22} & \cdots & 0 \\ \cdot & \cdot & \cdot & & \\ \cdot & \cdot & \cdot & & \\ \cdot & \cdot & \cdot & & \\ \cdot & \cdot & \cdot & & \\ \phi_{n0} & \phi_{n1} & \phi_{n2} & \cdots & \phi_{nn} \end{bmatrix} \begin{bmatrix} p_0(t_m = 0) \\ p_1(t_m = 0) \\ p_2(t_m = 0) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ p_n(t_m) = 0 \end{bmatrix}$$

where

$$\phi_{Ki} = \prod_{x=1}^{k-1} (n - x) \sum_{r=1}^{K} \left[\frac{e^{-(n - r)\lambda t_m}}{\prod_{\substack{y=1 \\ y \neq r}}^{k} (r - y)} \right] \qquad (2\text{-}74)$$

Applying these equations to a two-element redundant system with module failure rate λ, the author obtains

$$p_0(t_m) = e^{-2\lambda t_m} p_0(0) \qquad (2\text{-}75)$$

$$p_1(t_m) = \left[2e^{-\lambda t_m} - 2e^{-2\lambda t_m}\right] p_0(0) + e^{-\lambda t_m} \qquad (2\text{-}76)$$

$$p_2(t_m) = \left[1 - 2e^{-\lambda t_m} + e^{-2\lambda t_m}\right] p_0(0) + \left[1 - e^{-\lambda t_m}\right] p_1(0) + 1p_2(0) \qquad (2\text{-}77)$$

The probability of the system being available, p_A, (i.e., above the defined threshold of system failure) upon reaching the steady state condition during the standby interval, is given by

$$p_A = \sum_{K=0}^{q} p_K(t_s = \infty) \qquad (2\text{-}78)$$

The probability of being in state K at the end of the standby interval is

$$p_K(t_s = \infty) = \frac{1}{\sum_{J=0}^{n} \left(\frac{\lambda}{\mu}\right)^{J-K} \frac{(n - K)!}{(n - J)!}} \quad \text{for } K = 0, 1, \ldots, n \qquad (2\text{-}79)$$

Finally, the expected system downtime per year is given by

$$DT/y_r = 8760(1 - p_A) \text{ hours/year} \qquad (2\text{-}80)$$

The author also presents two sets of curves; one set shows the probability of mission failure as a function of n, MTBF/MTTR (mean time to repair) and t_m/MTBF for different values of q. The second set of curves give the expected system donwtime per year as a function of n, q, and MTBF/MTTR. These curves are helpful in gaining insight regarding the tradeoffs among the various system parameters. Several other examples of Markov process applications to the reliability analysis of redundant systems were found in the literature. In most cases, these additional applications are extensions or modifications of the examples already presented. The interested reader may refer to Sandler (Ref 73) for an extensive general discussion of the application of Markov processes to the reliability analysis of various system configurations.

Maintenance Systems. Brosh (Ref 16) utilizes a Markovian model to analyze a multi-service maintenance system with "first come, first served" repair (queue) discipline. The system is inspected at fixed periods of time, classified into one of a finite number of decisions. The state of the system is defined by the number of machines in the system, $i = 0,1,\ldots,N$, where N is the maximum number of machines the system can contain. There is only one maintenance crew. However, repairs may be performed in two different ways. This results in two different probabilities of

completing maintenance in a unit of time. A set of costs is associated with being in each state and using each type of service. An additional cost is incurred when a machine arrives for service and the system is full. The author presents a procedure for choosing the optimal policy for controlling the system by deriving the steady-state transition probabilities. Maintenance policies are considered by partitioning the state space into two sets - one containing those states in which the unit is maintained with service type two. The measure of effectiveness is the expected average cost per observation period over an indefinite sequence of observations. Analysis of the maintenance policy space reveals the interrelationship between system variables and optimal maintenance policies; e.g., disconnected policies are dominated by connected policies except for one case $(c_2)/(c_1) = (\mu_2/\mu_1)$ and $(c_3/c_1) = (1)/(\mu_1)$ for which any policy chosen will yield the same value $(c_1, (\lambda)/(\mu_1))$ for the cost function (Ref 16:78).

Natarajan (Ref 64) considers another important aspect of the system maintenance problem - that of assigning repair priority to a particular type of component. The author presents a Markovian characterization of an anti-aircraft system consisting of redundant radars working in conjunction with redundant computers. System failure occurs only when both radars or both computers are in a failed condition. As soon as any component fails, repairs are initiated. If, in the interim, a component of the other parallel system fails, it must either wait for repairs or it can preempt the unit in

repair. The first case is known as first-in, first-out (FIFO) discipline; the second case is known as preemptive discipline. The author assumes exponential distributions for the component failure rates (λ_1 and λ_2) and repair rates (μ_1 and μ_2). He derives somewhat lengthy expressions for mean time to system failure for both the FIFO repair policy and the preemptive policy. When no repairs are permitted, the mean time to system failure is

$$E(T) = \frac{1}{2(\lambda_1 + \lambda_2)} \left\{ 5 - \frac{2\lambda_1}{\lambda_1 + \lambda_2} - \frac{2\lambda_2}{2\lambda_1 + \lambda_2} \right\} \qquad (2\text{-}81)$$

The author discusses the problem of optimal priority assignments to which component and what priority and compares variations exhibited by the mean time to system failure with a priority discipline imposed on the repair process of different components. Numerical results provide the author with the following conclusions:

(1) FIFO discipline for repair of priority components is more effective than a preemptive discipline when $\lambda_1 > \lambda_2$ and $\mu_1 > \mu_2$ or when $\lambda_1 < \lambda_2$ and $\mu_1 < \mu_2$. Hence, the preemptive disciplines should be used only under special circumstances governed by emerging or risk considerations.

(2) When a preemptive discipline is imposed on the repair process, higher mean time to system failure will result if $\lambda_1 > \lambda_2$ and $\mu_1 < \mu_2$ (Ref 64:107).

As a final example of the use of Markov processes in a maintenance environment, Eckles (Ref 26) discusses a system

that deteriorates stocastically and is further complicated by the fact that the state of the system is unobservable unless an inspection is performed. The author develops a model to determine the appropriate sequence of actions (replacements, repairs, inspections, etc.) that minimizes the total cost of operation (including such factors as downtime, inefficiency, and salvage value). Maintenance of the system is characterized by a discrete-parameter, non-stationary Markov process. Prior to each transition, the decision maker selects one of a finite number of available actions. The action selected and the system's age determine the subsequent one-step transition probabilities and the conditional (on the system states) distribution of the next measurements. Cost is dependent on the action taken and on the system's state assigned to each possible transition. The author shows that the action that minimizes the discounted value of expected immediate and future costs (assuming optimum future actions) is determined by the system's age and the posterior distribution over the states (Ref 26:16). Optimum maintenance policies can then be calculated using a dynamic programming method.

Availability. The expression for system availability, A(t), integrates the probabilities of reliability and maintainability. A(t) may be defined as the probability that a predicted percentage of operations of time duration T will not have any malfunctions which cannot, through maintainability,

be restored to service within the permissible repair time constraint (Ref 5:8). Figure 10 shows the basic availability graph for a single component system. The differential equations for each state probability may be written directly from the figure (Ref 58:33).

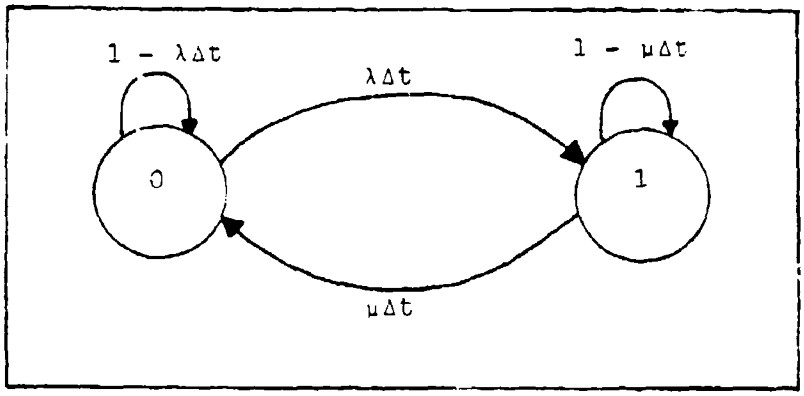


Fig. 10

Availability Graph for a Single Component System

$$\begin{bmatrix} p_0'(t) \\ p_1'(t) \end{bmatrix} = \begin{bmatrix} -\lambda & \mu \\ \lambda & -\mu \end{bmatrix} \begin{bmatrix} p_0(t) \\ p_1(t) \end{bmatrix} \tag{2-82}$$

Assuming the initial conditions $p_0(0) = 1$ and $p_1(0) = 0$, the solution of equation (2-82) yields for state probabilities

$$p_0(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\mu + \lambda} e^{-(\lambda + \mu)t} \tag{2-83}$$

$$p_1(t) = 1 - p_0(t) = \frac{\lambda}{\mu + \lambda} - \frac{\lambda}{\mu + \lambda} e^{-(\lambda + \mu)t} \tag{2-84}$$

The availability of the single component is the probability that the system is in state zero, i.e., $A(t) = p_0(t)$. For large t, the availability approaches a steady state value

$$A_{ss} = \lim_{t \to \infty} A(t) = \frac{\mu}{\lambda + \mu} \qquad (2\text{-}85)$$

noting that $\lambda = 1/(MTBF)$ and $\mu = (1)/(MTTR)$, A_{ss} can be re-written as

$$A_{ss} = \frac{MTBF}{MTBF + MTTR} \qquad (2\text{-}86)$$

Kapur (Ref 43:227) attained the same results for the availability as the above equations by stating that for the exponential distribution and for some interval of time Δt, he stated

$$p[\text{system failure during } \Delta t] = \lambda \Delta t \qquad (2\text{-}87)$$

and

$$p[\text{repair during } \Delta t/\text{system failure}] = \mu \Delta t \qquad (2\text{-}88)$$

He defines the availability function, A(t), by using equations (2-87) and (2-88) as follows

$$A(t+\Delta t) = A(t)(1 - \lambda\Delta t) + [1 - A(t)]\mu\Delta t$$

$$= A(t) - \lambda A(t)\Delta t + \mu\Delta t - \mu A(t)\Delta t$$

or

$$\frac{A(t+\Delta t) - A(t)}{\Delta t} = -(\lambda + \mu)A(t) + \mu$$

Taking the limit as $\Delta t \to 0$

$$\frac{\partial}{\partial t} A(t) = -(\lambda + \mu)A(t) + \mu$$

$$A'(t) = -(\lambda + \mu)A(t) + \mu$$

which is a differential equation whose solution is

$$A(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\mu + \lambda} e^{-(\lambda + \mu)t}$$

$$A = \lim_{t \to \infty} A(t) = \frac{\mu}{\lambda + \mu}$$

$$A = \frac{\text{Mean repair rate}}{\text{Mean failure rate + Mean repair rate}} \quad (2\text{-}89)$$

or equivalently

$$A = \frac{\text{Mean time to failure}}{\text{Mean time to repair + Mean time to failure}} \quad (2\text{-}90)$$

He stated that, fortunately, it so happens that if the downtime, p. d. f., is taken as something other than exponential, the same steady state solution results. In practice, frequently the log normal is used as the downtime, p. d. f. Kapur defined the intrinsic availability in the same way as the availability. The only difference will be that the mean repair rate in the availability function will be replaced by the mean active repair rate. Thus the steady state intrinsic availability is given by

$$A_I = \frac{\text{Mean time to failure}}{\text{Mean active repair time + Mean time to failure}} \quad (2\text{-}91)$$

Dayon (Ref 23) utilizes the steady-state availability concept to analyze a computer system consisting of a data processor and tape units. The purpose of the analysis is to solve for the MTTR of the redundant system. The author points out that defining the system states and formulating the appropriate system steady-state availability transition rate diagram is the step requiring the greatest degree of ingenuity and expertise. By contrast, subsequent steps to obtain a numerical solution for the system MTTR involves only routine mathematical manipulations (Ref 23:153). The system under discussion consists of a data processor (unit #1) requiring two tape units (units #2) for data storage. A third tape unit is on standby redundancy. A block diagram of the system is shown in Figure 11. Implied in the model, but not explicitly stated, are the assumptions of a Markovian tape system with constant element failure and repair rates. Other features of the system include:

(1) All three tape units are identical.

(2) Any tape unit can be removed off-line and repaired while the system is energized.

(3) All units are de-energized when the system enters a fail state.

(4) Repair is performed on a FIFO basis.

(5) The system is re-energized and operated as soon as there are enough units repaired to have full system capability.

(6) There is one repairman.

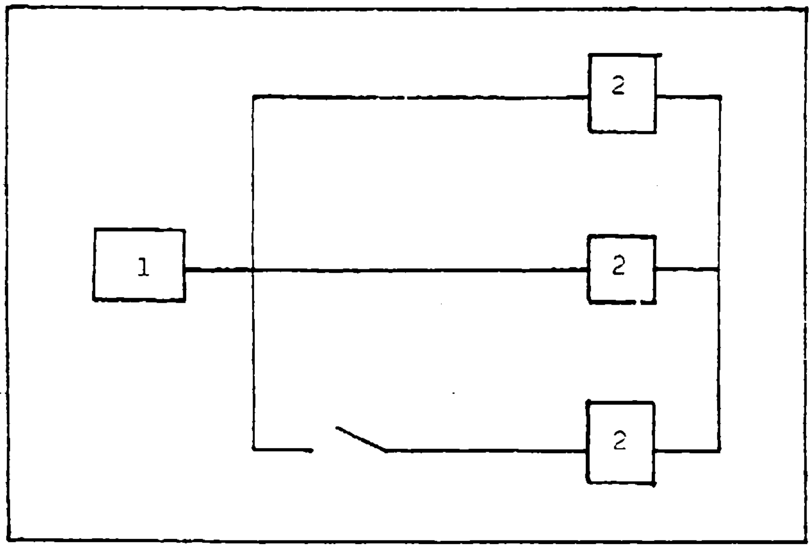


Fig. 11

Data Processor (#1) and Tape Units (#2)

The assumed failure and repair rates for the respective units are:

λ_1 = 0.01/hour μ_1 = 1.0/hour

λ_2 = 0.02/hour μ_2 = 2.0/hour

The states of the system are defined as:

State 0: all the units up

State 1: one tape unit down

State f_1: two tape units down

State f_2: one tape and the data processor down

State f_3: data processor down

The steady state availability transition-rate diagram for this system is shown in Figure 12. The loops showing the probability of remaining in each state have been omitted

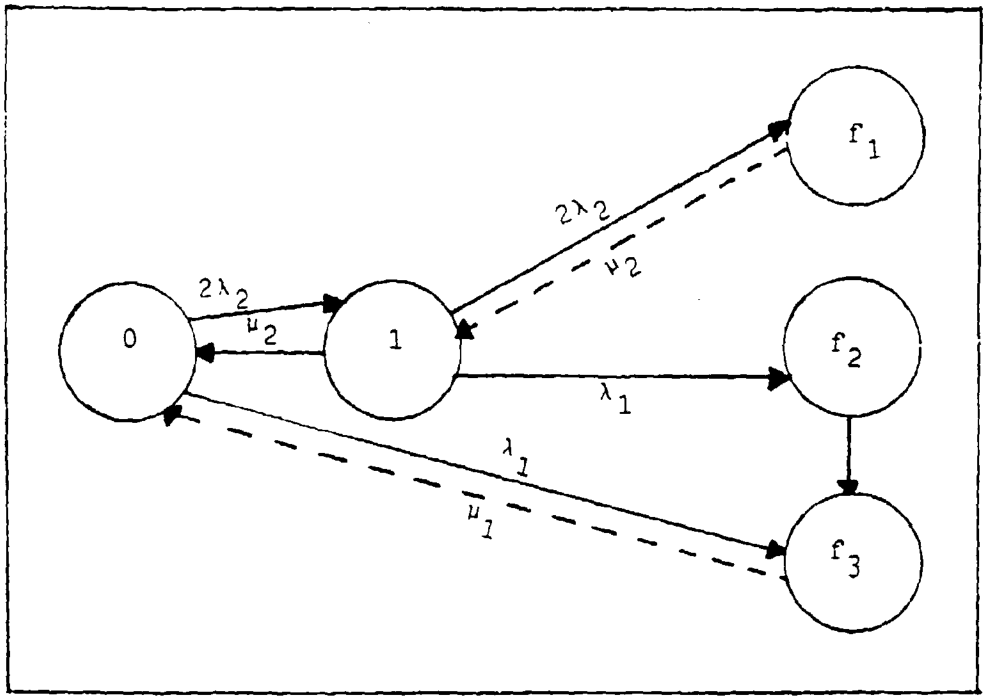


Fig. 12

Transition-rate Diagram for Computer System

to enhance clarity. The repair transition from state f_2 to state f_3 is indicative of the FIFO repair discipline. If the policy had been to resume operation as soon as possible, the repairman would have ceased work on the tape unit when the system entered f_2 and performed maintenance on the data processor until the system was returned to state 1. The author develops the availability matrix and substitutes the appropriate failure and repair rates. Numerical solution yields (after "normalizing" the state probabilities) a value for MTTR equal to 1.46 hours.

$$
\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -0.5 & 2.0 & 0 & 0 & 1.0 \\ 0.04 & -2.05 & 2.0 & 0 & 0 \\ 0 & 0.04 & -2.0 & 0 & 0 \\ 0 & 0.01 & 0 & -2.0 & 0 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_{f_1} \\ p_{f_2} \\ p_{f_3} \end{bmatrix} \qquad (2\text{-}92)
$$

Grace (Ref 34) discusses the evaluation of steady-state system availability when there are a limited number of repairable spares for the various types of on-line units. The particular system considered consists of five on-line units and two spares of a given type, with one maintenance man (associated with a replacement rate δ) and one repairman (associated with a repair rate μ) assigned to the unit type. Failure, repair, and replacement times are assumed to be exponentially distributed. Hence, the availability of i units of type a, A_{ai}, can be determined from a Markov model which includes all possible states and transitions of the on-line and spare units of type a. The author includes a complete transition diagram (33 states) and discusses a procedure for writing the resulting equations directly from the transition diagram. For this system, the steady-state probability Π_{ij} of being in state ij is obtained by solving the set of equations:

$$
\begin{aligned}
(n\lambda + s\alpha)\Pi_{00} &= \mu\Pi_{01} + \delta\Pi_{1,\,-1} \\
[n\lambda + (s-1)\alpha]\Pi_{01} &= \mu\Pi_{02} + \delta\Pi_{1,\,0} \qquad (2\text{-}93) \\
&\text{etc.}
\end{aligned}
$$

Where n is the number of on-line positions, α is the off-line (spare) failure rate, and s is the number of spares. The normalizing condition

$$\sum_{i,j} \Pi_{ij} = 1 \qquad (2\text{-}94)$$

is required for a unique solution. The steady-state probability that i on-line units are not operating is given by

$$\Pi_i = \sum_j \Pi_{ij} \qquad (2\text{-}95)$$

If i units have failed, the probability that any l units are good is

$$p\ell_i = \frac{\binom{n-\ell}{i}\Pi_i}{\binom{n}{i}} = \frac{\binom{n-i}{\ell}\Pi_i}{\binom{n}{\ell}} \qquad (2\text{-}96)$$

The author shows that the total probability that any l particular units are working is

$$Aa_\ell = \sum_i p\ell_i = \sum_i \frac{\binom{n-\ell}{i}\Pi_i}{\binom{n}{i}} \qquad (2\text{-}97)$$

Schick (Ref 76) employs a Markov process to determine the availability or "operational readiness" of a system which is subject to inspection and repair. Except for inspection and repair periods, the system is kept in a normal mode from which it is called if its operation is required. Failure of a primary part causes immediate shutdown, inspection, and

repair. Failure during checkout is detectable only at periodic inspections while failure of some equipment is liable to two types of failure. Examples of such systems include missile launch facilities and computer repair systems.

Grippo (Ref 37) also employs a Markov process to evaluate the reliability and availability of large, complex systems. The author's approach enables the analyst to introduce system operation or maintenance constraints without adding appreciably to the complexity of the solution. Algorithms are derived for obtaining computer-aided transient and steady-state solutions to both reliability and availability problems. The author also presents a unique method for integrating the differential equations that can result from a Markov process.

Sasaki (Ref 74) utilizes implied Markov process techniques to develop a set of charts and nomographs which can be used to evaluate trade-off characteristics between system reliability and maintainability. The author presents a slide methods (using the charts and nomographs) of achieving desired values of system (mission) availability for duplex parallel redundancy and duplex switch over redundancy.

Finally, Hevesh and Harrahy (Ref 38) discuss the effect of failure on the availability or "readiness" of phased, array radar systems under different structural and repair capacity conditions. The author also develops expressions for the availability of parallel operated equipment, whether truly or quasi-redundant.

III. Summary of Point Estimates of Availability, Reliability of Maintained Systems

Computation of Reliability and Availability

The techniques for obtaining the reliability and availability of a repairable structure with state dependent hazards and repair rates is best illustrated by an example (Ref 58). Consider the following two components parallel system with:

components x_1 and x_2

components hazards λ_1, λ_2

repair rates μ_1, μ_2

number of repairmen: 1

In general, five states are needed as tabulated in Table 3.1.

Table 3.1

System States for 2 Component Repairable System

Good States	Bad States
0 - x_1, x_2	3 - $\bar{x}_1$, $\bar{x}_2$ (x_1 failed before x_2)
1 - $\bar{x}_1$, x_2	4 - $\bar{x}_2$, $\bar{x}_1$ (x_2 failed before x_1)
2 - x_1, $\bar{x}_2$	

The transition diagrams for calculating the system reliability and availability are given in Figure 13. The distinction between the availability and reliability transition diagrams is that the availability transition diagram repairs are

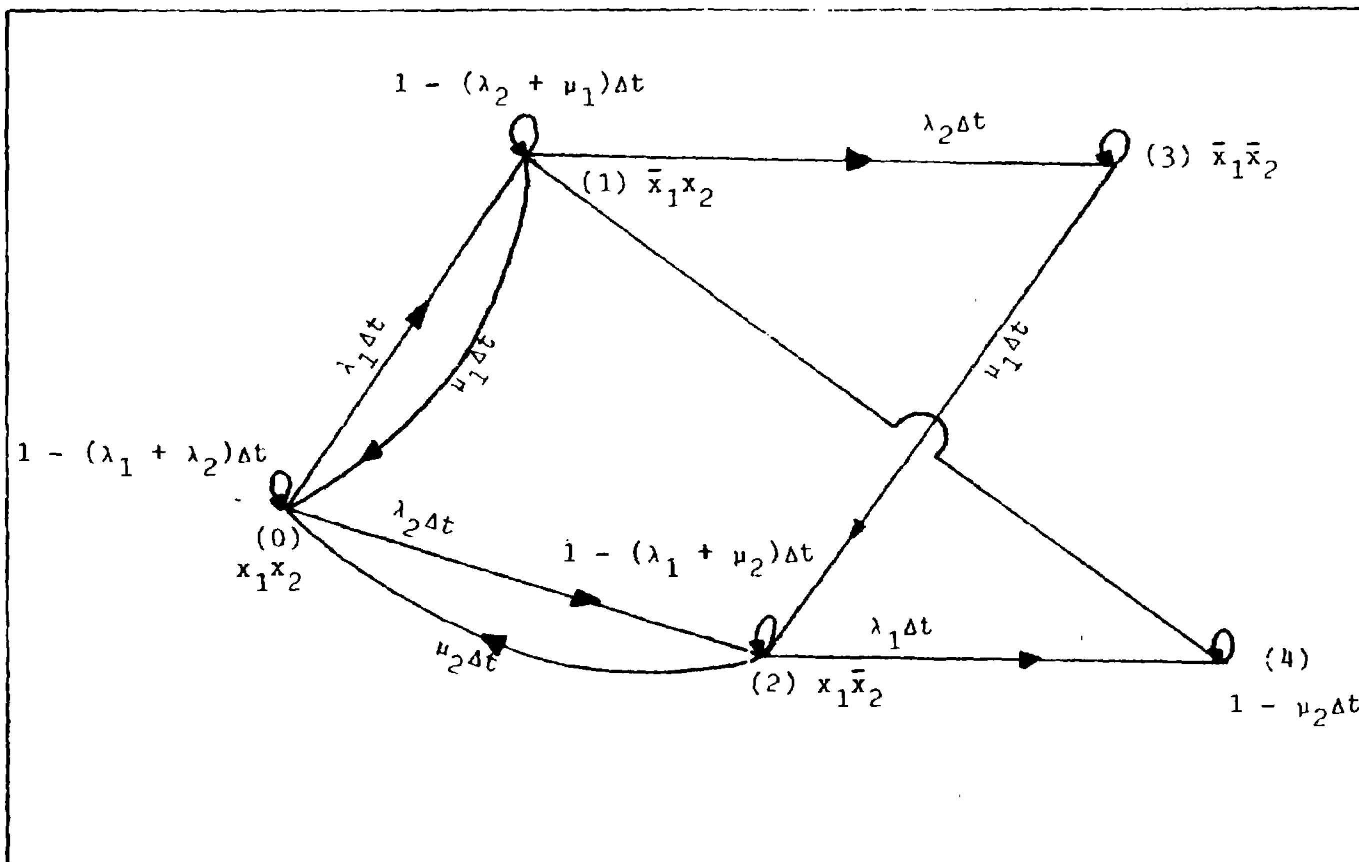


Fig. 13 Transition Diagrams for Two Component System

(a) Availability

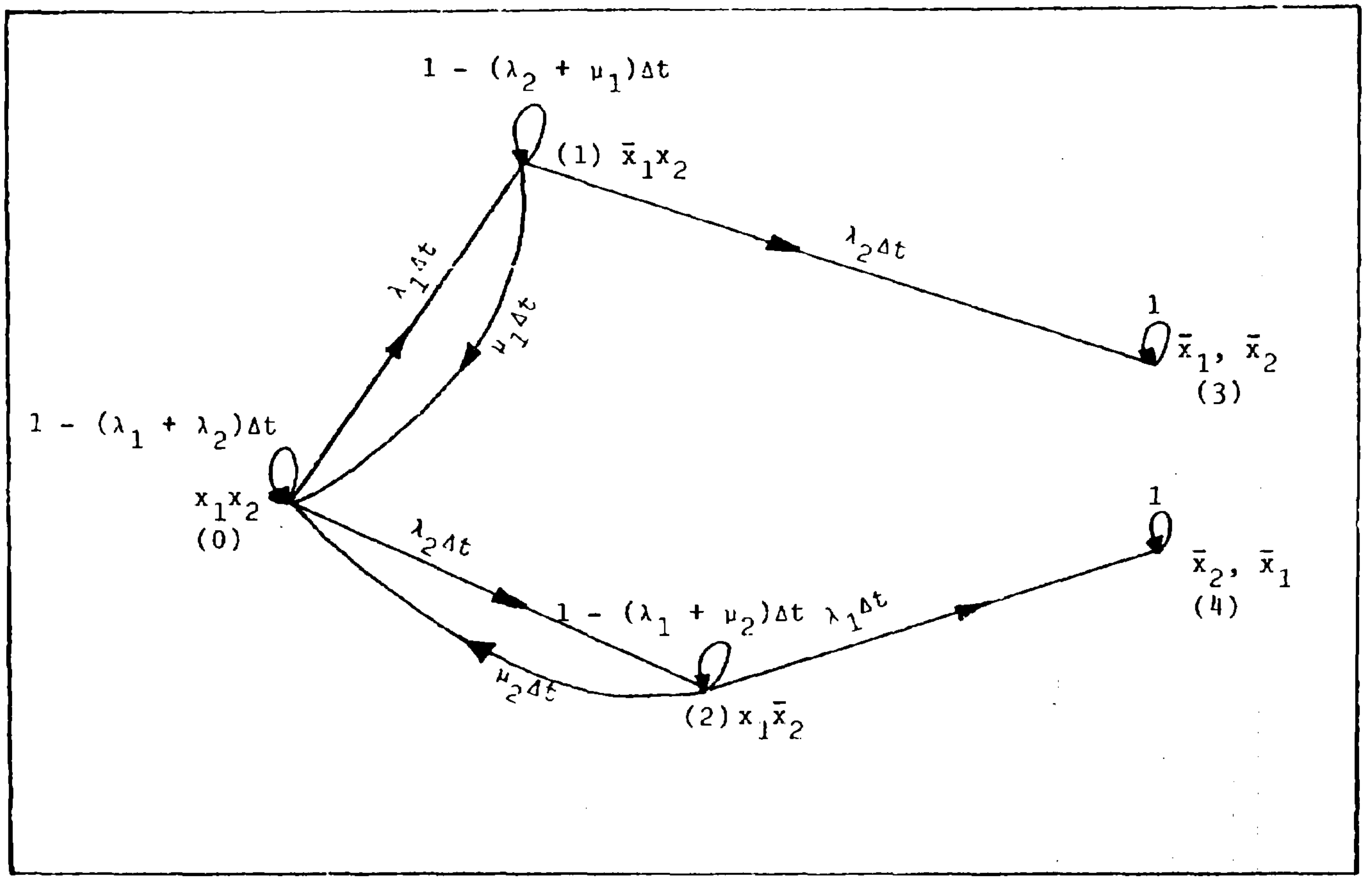


Fig. 13 Transition Diagrams for Two Component System (continued)

(b) Reliability

allowed from all system states, while for the reliability transition diagram repairs are allowed only from the good system states. Therefore, summing the good state probabilities obtained from the availability diagram gives the probability that the system is good at time t, while summing the good state probabilities obtained from the reliability transition diagram gives the probability that the system has never entered a failed state. System states that distinguish the order in which component failures occur are necessary for the availability model since there is only one repairman, and according to the repair policy, he will work on the component that fails first. If the system is in state number three $(\bar{x}_1, \bar{x}_2)$, then the repairman is working on component x_1 and the next transition must be to state number two $(x_1, \bar{x}_2)$. If the system is in state number four $(\bar{x}_2, \bar{x}_1)$, then the repairman is working on component x_2 and the next transition will be to state one $(\bar{x}_1, x_2)$. Ordering of the component failures is not necessary for the reliability transition diagram since once the system enters a state with more than one failure, no repair is attempted.

The differential equations for the state probabilities are given for availability analysis by

$$\begin{bmatrix} p_0'(t) \\ p_1'(t) \\ p_2'(t) \\ p_3'(t) \\ p_4'(t) \end{bmatrix} = \begin{bmatrix} -(\lambda_1 + \lambda_2) & \mu_1 & \mu_2 & 0 & 0 \\ \lambda_1 & -(\lambda_2 + \mu_1) & 0 & 0 & \mu_2 \\ \lambda_2 & 0 & -(\lambda_1 + \mu_2) & \mu_1 & 0 \\ 0 & \lambda_2 & 0 & -\mu_1 & 0 \\ 0 & 0 & \lambda_1 & 0 & -\mu_2 \end{bmatrix} \begin{bmatrix} p_0(t) \\ p_1(t) \\ p_2(t) \\ p_3(t) \\ p_4(t) \end{bmatrix} \tag{3.1a}$$

where

$$A(t) = p_0(t) + p_1(t) + p_2(t) \tag{3.1b}$$

and for the reliability analysis by

$$\begin{bmatrix} p_0'(t) \\ p_1'(t) \\ p_2'(t) \\ p_3'(t) \\ p_4'(t) \end{bmatrix} = \begin{bmatrix} -(\lambda_1 + \lambda_2) & \mu_1 & \mu_2 & 0 & 0 \\ \lambda_1 & -(\lambda_2 + \mu_1) & 0 & 0 & 0 \\ \lambda_2 & 0 & -(\lambda_1 + \mu_2) & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 & 0 \\ 0 & 0 & \lambda_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_0(t) \\ p_1(t) \\ p_2(t) \\ p_3(t) \\ p_4(t) \end{bmatrix} \tag{3.2a}$$

where

$$R(t) = p_0(t) + p_1(t) + p_2(t) \tag{3.2b}$$

Ordering would not have been necessary if there were two repairmen. The availability transition diagram is given in Figure 14. The reliability transition diagram remains uncharged.

If the components are identical ($\lambda_1 = \lambda_2 = \lambda$) and ($\mu_1 = \mu_2 = \mu$), then states 1 and 2 can be combined and states 3 and 4 can be combined resulting in a greatly simplified

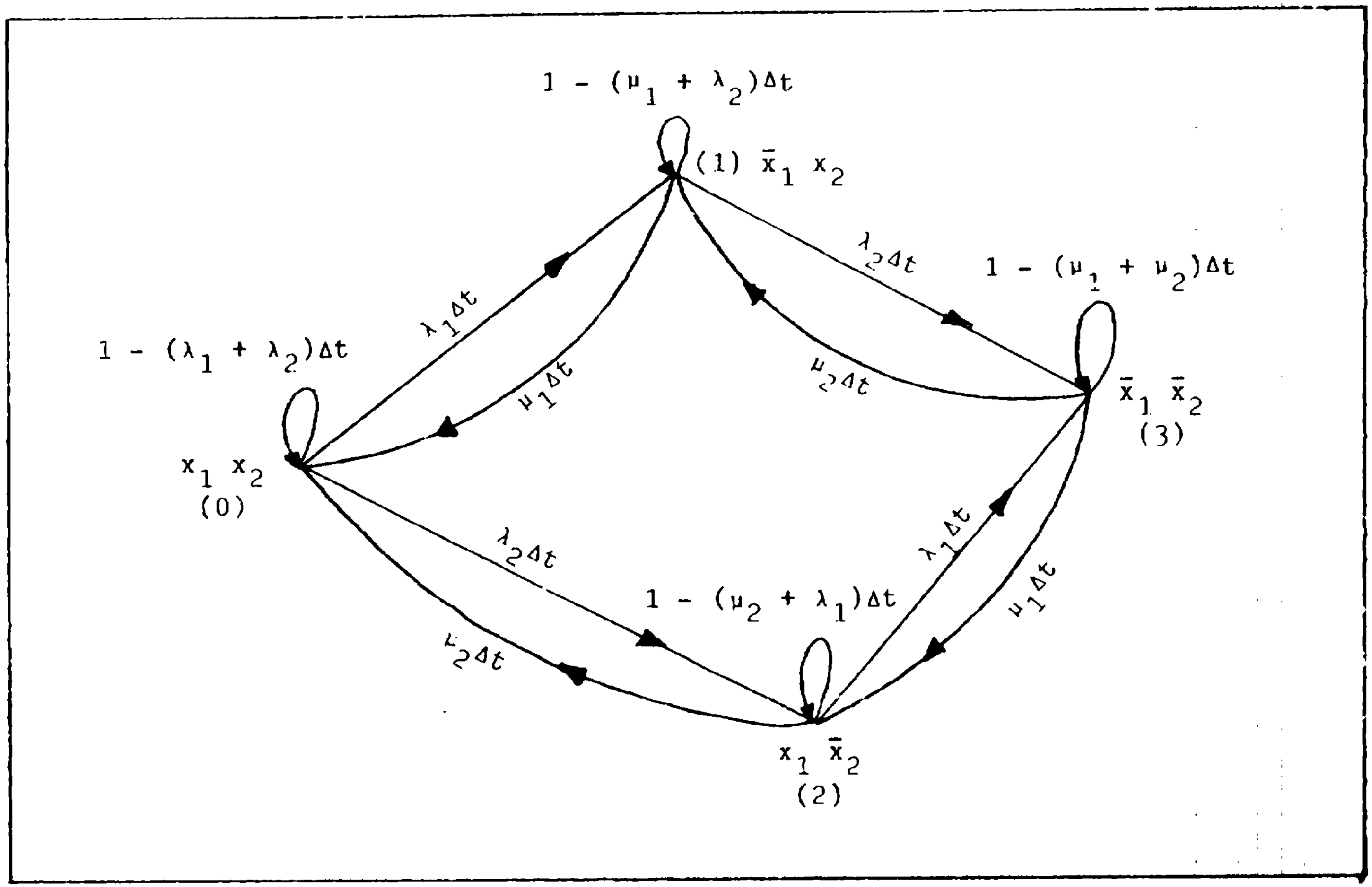


Fig. 14

Availability Transition Diagram for Two Component System

with Two Repairmen (No Joint Effort)

transition diagram as in Figure 15.

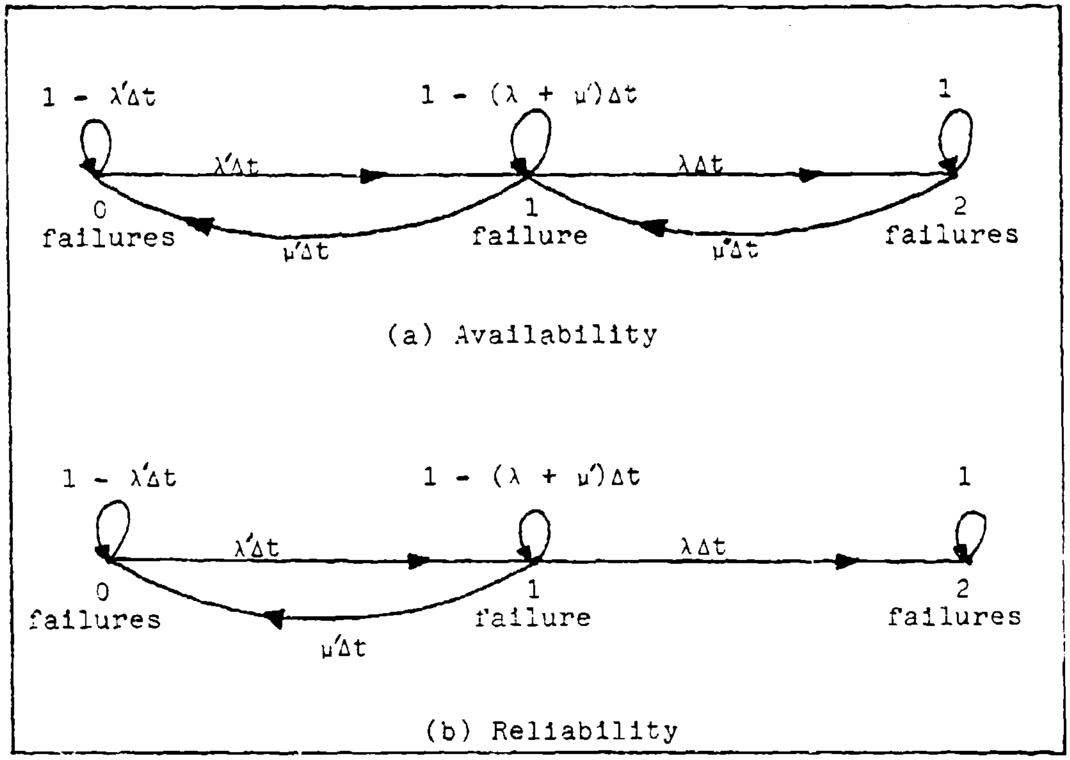


Fig. 15

Transition Diagrams for General System of Two Identical Components

It should be noted that for the case of two repairmen, when one component is failed one has the option of assigning only one of the repairmen or assigning both repairmen permitting joint effort. When a joint effort is permitted, the repair rate $\mu' = \beta\mu$. Sandler (Ref 73) uses $\beta = 1.5$.

The differential equations for system availability is, therefore, given by

$$\begin{bmatrix} p_0'(t) \\ p_1'(t) \\ p_2'(t) \end{bmatrix} = \begin{bmatrix} -\lambda' & \mu' & 0 \\ \lambda' & -(\lambda + \mu') & \mu'' \\ 0 & \lambda & -\mu'' \end{bmatrix} \begin{bmatrix} p_0(t) \\ p_1(t) \\ p_2(t) \end{bmatrix} \quad (3\text{-}3a)$$

where

$$A(t) = p_0(t) + p_1(t) \quad (3\text{-}3b)$$

and the differential equations for a system reliability is given by

$$\begin{bmatrix} p_0'(t) \\ p_1'(t) \\ p_2'(t) \end{bmatrix} = \begin{bmatrix} -\lambda' & \mu' & 0 \\ \lambda' & -(\lambda + \mu') & 0 \\ 0 & \lambda & 0 \end{bmatrix} \begin{bmatrix} p_0(t) \\ p_1(t) \\ p_2(t) \end{bmatrix} \quad (3\text{-}4a)$$

where

$$R(t) = p_0(t) + p_1(t) \quad (3\text{-}4b)$$

Equation (3-3) is readily solved for the system availability yielding

$$A(t) = 1 - \frac{\lambda\lambda'}{r_1 r_2} - \frac{\lambda\lambda'}{r_1 - r_2}\left[\frac{e^{r_1 t}}{r_1} - \frac{e^{r_2 t}}{r_2}\right] \quad (3\text{-}5)$$

where r_1 and r_2 are the roots of the equation

$$r^2 + (\lambda + \lambda' + \mu' + \mu'')r + (\lambda\lambda' + \lambda'\mu'' + \mu'\mu'') = 0$$

where

$$\begin{aligned} r_1 + r_2 &= -(\lambda + \lambda' + \mu' + \mu'') \\ r_1 r_2 &= \lambda\lambda' + \lambda'\mu'' + \mu'\mu'' \end{aligned} \quad (3\text{-}6)$$

The steady state availability is given by

$$A_{ss} = \lim_{t \to \infty} A(t) = 1 - \frac{\lambda\lambda'}{r_1 r_2}$$

$$= \frac{\lambda'\mu'' + \mu'\mu''}{\lambda\lambda' + \lambda'\mu'' + \mu'\mu''} \qquad (3\text{-}7)$$

Equation (3-4) is readily solved for system reliability yielding

$$R(t) = \frac{1}{r_3 r_4}\left[r_3 e^{r_4 t} - r_4 e^{r_3 t}\right] \qquad (3\text{-}8)$$

where r_3 and r_4 are the solutions of

$$r^2 + (\lambda + \lambda' + \mu')r + \lambda\lambda' = 0 \qquad (3\text{-}9)$$

The roots r_3 and r_4 are clearly negative real since the discriminant of equation (3-9) satisfies

$$b^2 - 4ac = (\lambda + \lambda' + \mu')^2 - 4\lambda\lambda'$$

$$= (\lambda - \lambda')^2 + 2\mu'(\lambda + \lambda') + \mu'^2 > 0$$

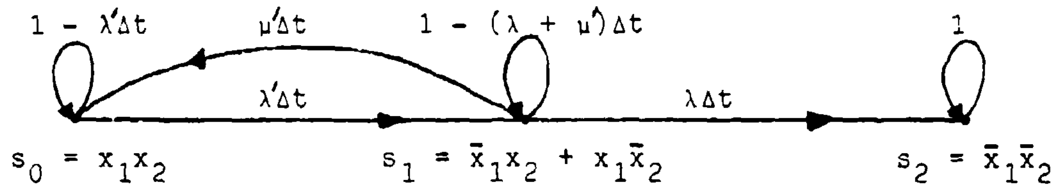
Shooman (Ref 78) gave the solution for the two identical parallel elements and K repairmen. He derived the following equations with respect to Figure 16. The differential equations associated with Figure 16 are

$$p'_{s0}(t) + \lambda p_{s0}(t) = \mu' p_{s1}(t)$$

$$p'_{s1}(t) + (\mu' + \lambda)p_{s1}(t) = \lambda' p_{s0}(t) \qquad (3\text{-}10)$$

$$p'_{s2}(t) = \lambda p_{s1}(t)$$

$$p_{s0}(0) = 1 \quad , \quad p_{s1}(0) = p_{s2}(0) = 0$$



$\lambda' = 2\lambda$ for an ordinary system

$\lambda' = \lambda$ for a standby system

$\mu' = \mu$ for one repairman

$\mu' = K\mu$ for more than one repairman $(K > 1)$

Fig. 16

Markovian Reliability Model for Two Identical Parallel Elements and K Repairmen

Taking the laplace transform of this set of equations yields

$$\begin{aligned} (s + \lambda')p_{s0}(s) - \mu' p_{s1}(s) &= 1 \\ -\lambda' p_{s0}(s) + (s + \mu' + \lambda)p_{s1}(s) &= 0 \\ -\lambda p_{s1}(s) + sp_2(s) &= 0 \end{aligned} \quad (3\text{-}11)$$

Solving the set of equations of (3-11) using Cramer's Rule yields

$$\begin{aligned} p_{s0}(s) &= \frac{(s + \lambda + \mu')}{s^2 + (\lambda + \lambda' + \mu')s + \lambda\lambda'} \\ p_{s1}(s) &= \frac{\lambda'}{s^2 + (\lambda + \lambda' + \mu')s + \lambda\lambda'} \\ p_{s2}(s) &= \frac{\lambda\lambda'}{s[s^2 + (\lambda + \lambda' + \mu')s + \lambda\lambda']} \end{aligned} \quad (3\text{-}12)$$

Solution for the roots of the denominator quadratic yields

$$r_1, r_2 = \frac{-(\lambda + \lambda' + \mu') \pm \sqrt{(\lambda + \lambda' + \mu')^2 - 4\lambda\lambda'}}{2} \quad (3\text{-}13)$$

Expanding equation (3-12) in partial fractions

$$p_{s0}(s) = \frac{s + \lambda + \mu'}{(s - r_1)(s - r_2)}$$

i.e.,

$$p_{s0}(s) = \frac{(\lambda + \mu' + r_1)/(r_1 - r_2)}{(s - r_1)} + \frac{(\lambda + \mu' + r_2)/(r_2 - r_1)}{(s - r_2)} \quad (3\text{-}14)$$

$$p_{s1}(s) = \frac{\lambda'}{(s - r_1)(s - r_2)} = \frac{\lambda'/(r_1 - r_2)}{s - r_1} + \frac{\lambda'/(r_2 - r_1)}{s - r_2} \quad (3\text{-}15)$$

$$p_{s2}(s) = \frac{\lambda\lambda'}{s(s - r_1)(s - r_2)}$$

$$= \frac{\lambda\lambda'/r_1 r_2}{s} + \frac{\lambda\lambda'/r_1(r_1 - r_2)}{s - r_1} + \frac{\lambda\lambda'/r_2(r_2 - r_1)}{s - r_2} \quad (3\text{-}16)$$

As a check one can sum equations (3-14) to (3-16) noting that $r_1 r_2 = \lambda\lambda'$ and $r_1 + r_2 = -(\lambda + \lambda' + \mu')$. The result is

$$p_{s0}(s) + p_{s1}(s) + p_{s2}(s) = \frac{1}{s} \quad (3\text{-}17)$$

Taking the inverse transforms, one obtains

$$p_{s0}(t) + p_{s1}(t) + p_{s2}(t) = 1$$

The inverse transforms of equations (3-14) to (3-16) yield

$$p_{s0}(t) = \frac{\lambda + \mu' + r_1}{r_1 - r_2}\, e^{r_1 t} - \frac{\lambda + \mu' + r_2}{r_1 - r_2}\, e^{r_2 t} \quad (3\text{-}18)$$

$$p_{s1}(t) = \frac{\lambda'}{r_1 - r_2} e^{r_1 t} - \frac{\lambda'}{r_1 - r_2} e^{r_2 t} \qquad (3\text{-}19)$$

$$p_{s2}(t) = 1 + \frac{r_2}{r_1 - r_2} e^{r_1 t} - \frac{r_1}{r_1 - r_2} e^{r_2 t} \qquad (3\text{-}20)$$

From equation (3-13) r_1 and r_2 are always negative real numbers; therefore, the time functions are decaying exponentials. For a system composed of two series elements, the system reliability is unaffected by repair, and $R(t) = e^{-2\lambda t}$, which can be obtained by setting $\mu' = 0$ in the expression for $p_{s0}(t)$. For a parallel system (standby or ordinary), the reliability is given by $p_{s0}(t) + p_{s1}(t)$. By appropriately choosing coefficients λ', μ', and μ'' (as given in Table 3-2), a large number of systems can be modeled.

Table 3.2 Coefficients of λ', μ', μ'' (Ref 58)

Repair/Non-repairable	Type	Repair Crew	λ'	μ'	μ''
non-repairable	parallel	0	2λ	0	0
non-repairable	standby	0	λ	0	0
repairable	parallel	1	2λ	μ	μ
repairable	parallel	2, no joint effort	2λ	μ	2μ
repairable	parallel	2, joint effort	2λ	$\beta\mu$	2μ
repairable	standby	1	λ	μ	μ
repairable	standby	2, no joint effort	λ	μ	2μ
repairable	standby	2, joint effort	λ	$\beta\mu$	2μ

Systems with more than two components are treated also by Messenger (Ref 58).

IV. Confidence Limits for Availabilities of Maintained Systems Exact Analytical Methods

An estimate of system availabilty calculated from time-to-failure and time-to-repair test data will be subject to some degree of uncertainty due to the uncertainty associated with the sample estimates of MTTF and MTTR. This chapter presents techniques for determining a lower confidence limit on system availability when time-to-failure and time-to-repair are independent, exponentially distributed variables. Also, the case of log normally distributed repair times will be included.

The Case of Both Exponential Distributions for the Time-to-Failure and Time-to-Repair

Mary Thompson (Ref 83) presents the techniques for determining a lower confidence limit by making use of the one-to-one correspondence between availability and the ratio of mean-time-to-repair and mean-time-to-failure in order to define F-distributed variables upon which the confidence limits are based.

The availability, as defined before, is usually defined as the probability that the system is operating satisfactorily at any point in time. This probability can be expressed mathematically as

$$A = \frac{\theta}{\theta + \phi} = \frac{1}{1 + (\phi/\theta)} \qquad (4-1)$$

where

θ = system mean-time-to-failure

ϕ = system mean-time-to-repair

The one-to-one correspondence between availability ϕ/θ is obvious. The usual sample estimate of availability is

$$\hat{A} = \frac{\hat{\theta}}{\hat{\theta} + \hat{\phi}} \qquad (4-2)$$

where $\hat{\theta}$, the sample estimate of θ, is calculated from

$$\hat{\theta} = \sum_{i=1}^{n_1} t_{1i}/n_1 \qquad (4-3)$$

where

t_{1i} = time between the (i - 1)th and the i-th failures

n_1 = number of failures

and $\hat{\phi}$, the sample estimate of ϕ, is calculated from

$$\hat{\phi} = \sum_{j=1}^{n_2} t_{2j}/n_2 \qquad (4-4)$$

where

t_{2j} = time-to-repair associated with the j-th failure

n_2 = number of repair actions initiated

It is assumed that t_1 (time-to-failure) and t_2 (time-to-repair) are stocastically independent random variables with probability density functions

$$f_1(t_1) = \frac{1}{\theta} e^{-t_1/\theta} \qquad (4-5)$$

and

$$f_2(t_2) = \frac{1}{\phi} e^{-t_2/\phi} \tag{4-6}$$

If we consider a random sample of n_1 times-to-failure and n_2 times-to-repair drawn from the above populations with random sample means $\hat{\theta}$ and $\hat{\phi}$ calculated from equations (4-3) and (4-4). It is well known that $2n_1\hat{\theta}/\theta$ and $2n_2\hat{\phi}/\phi$ are chi-square distributed variables with $2n_1$ and $2n_2$ degrees of freedom, respectively. Since they are independent due to the independence of the variables t_1 and t_2, it is possible to define two new variables:

$$z_1 = \left(\frac{2n_1\hat{\theta}/\theta}{2n_1}\right) \Bigg/ \left(\frac{2n_2\hat{\phi}/\phi}{2n_2}\right) = \frac{\hat{\theta}\phi}{\hat{\phi}\theta} \tag{4-7}$$

which is F-distributed with $2n_1$, $2n_2$ degrees of freedom, and

$$z_2 = \frac{1}{z_1} = \frac{\hat{\phi}\theta}{\hat{\theta}\phi} \tag{4-8}$$

which is F-distributed with $2n_2$, $2n_1$ degrees of freedom. The variable z_1 can be used to obtain a lower confidence limit for availability A as follows (Ref 83)

$$pr \left\{ \frac{\hat{\theta}\phi}{\hat{\phi}\theta} \le F_{1-\alpha}(2n_1, 2n_2) \right\} = 1 - \alpha$$

i.e.,

$$pr \left\{ \frac{\phi}{\theta} \le \frac{\hat{\phi}}{\hat{\theta}} F_{1-\alpha}(2n_1, 2n_2) \right\} = 1 - \alpha$$

$$\mathrm{pr}\left\{1 + \frac{\phi}{\theta} \le 1 + \frac{\hat{\phi}}{\hat{\theta}} F_{1-\alpha}(2n_1, 2n_2)\right\} = 1 - \alpha$$

$$\mathrm{pr}\left\{\frac{1}{1 + \frac{\hat{\phi}}{\hat{\theta}} F_{1-\alpha}(2n_1, 2n_2)} \le \frac{1}{1 + \frac{\phi}{\theta}}\right\} = 1 - \alpha$$

$$\mathrm{pr}\left\{\frac{\hat{\theta}}{\hat{\theta} + \hat{\phi} F_{1-\alpha}(2n_1, 2n_2)} \le A\right\} = 1 - \alpha \qquad (4-9)$$

Most practical cases $n_1 = n_2 = n$ and equation (4-9) becomes

$$\mathrm{pr}\left\{\frac{\hat{\theta}}{\hat{\theta} + \hat{\phi} F_{1-\alpha}(2n, 2n)} \le A\right\} = 1 - \alpha \qquad (4-10)$$

The $(1 - \alpha)$ lower confidence limits is found from

$$\mathrm{LCL} = \frac{\hat{\theta}}{\hat{\theta} + \hat{\phi} F_{1-\alpha}(2n, 2n)} \qquad (4-11)$$

A two-sided $(1 - \alpha)$ confidence interval, derived in a similar manner, is given by

$$\mathrm{LCL} = \frac{\hat{\theta}}{\hat{\theta} + \hat{\phi} F_{1-\alpha|2}(2n, 2n)} \qquad (4-12)$$

$$\mathrm{UCL} = \frac{\hat{\theta} F_{1-\alpha|2}(2n, 2n)}{\hat{\theta} F_{1-\alpha|2}(2n, 2n) + \hat{\phi}} \qquad (4-13)$$

Confidence intervals calculated from equations (4-11), (4-12), and (4-13) cover the true value of availability 100 $(1 - \alpha)$ percent of the time. The curves of the 0.9 and 0.95 lower confidence limits on availability calculated from equation (4-11) for values $(\hat{\phi}/\hat{\theta})$ ranging from 0 to 0.3 and for

for selected values of n between 2 and 50 are given in (Ref 83).

Suppose for example that during the field testing of a communication system, five failures were experienced. The average time-to-failure $\hat{\theta}$ was 125 hours. The average time-to repair $\hat{\phi}$ was 3 hours. Previous experience with similar systems indicates that time-to-failure and time-to-repair are exponentially distributed. Independence of the two variables assumed (in practical terminology, this means that the time required to fix a failure does not depend on how long the equipment operated prior to the failure) a point estimate of the system availability.

$$\hat{A} = \frac{\hat{\theta}}{\hat{\theta} + \hat{\phi}} = \frac{125}{125 + 3} = 0.9765$$

To find the 90 percent lower confidence limit, compute

$$\frac{\hat{\phi}}{\hat{\theta}} = \frac{3}{125} = 0.024$$

From the curves given (Ref 83), we can read directly the LCL on availability with $\hat{\phi}/\hat{\theta} = 0.024$ and $n = 5$; the 90 percent LCL is 0.95.

Lower Confidence Limits Assuming Lognormally Distributed Repair Times

Gray and Schucany (Ref 36) introduced the lower confidence limits in the case of lognormally distributed repair times using the tables of H. L. Gray and T. O. Lewis (Ref 35). If the random variables x and y denote the repair times and

times to failure, respectively, since the availability is defined by

$$A = \frac{\mu_y}{\mu_y + \mu_x} = \frac{1}{1 + \left(\frac{\mu_x}{\mu_y}\right)} \qquad (4\text{-}14)$$

where μ_x is the mean-time-to-repair (MTTR) and μ_y is the mean-time-between-failures (MTBF). It is difficult to work analytically with the assumption of lognormal x and exponential y in trying to establish confidence limits on the ratio of the means. Gray and Lewis (Ref 35) tabulate to some extent the distribution of the ratio of independent lognormal and chi-square quantities for known variance of the lognormal distribution. These tables enable us to establish a lower confidence limit for availability based on the assumption of lognormal repairs.

Gray and Schucany (Ref 36) derived the following LCL for the availability assuming lognormally distributed repair times. If x and y have lognormal and exponential distribution, the respective probability density functions are given by

$$h(x, \alpha, \beta^2) = \begin{cases} \dfrac{1}{\sqrt{2\pi}\, x\beta} \exp\left[-\dfrac{1}{2}\left(\dfrac{\ell n x - \alpha}{\beta}\right)^2\right], & x > 0 \\ 0 & \text{elsewhere} \end{cases} \qquad (4\text{-}15)$$

$$f(y; \mu_y) = \begin{cases} \dfrac{1}{\mu_y} \exp\left[-\dfrac{y}{\mu_y}\right], & y > 0 \\ 0 & \text{elsewhere} \end{cases} \qquad (4\text{-}16)$$

for the lognormal distribution, the parameter α and β^2 are the mean and variance of $\ell n(x)$, respectively; that is,

$$E[\ell nx] = \alpha$$

$$\mathrm{var}[\ell nx] = \beta^2$$

since

$$\mu_x = E[x] = \exp\,[\alpha + \tfrac{1}{2}\beta^2]$$

It can be seen that for a random sample of size n from $h(x;\ \alpha,\ \beta^2)$, the quantity defined by

$$Q_1 = \bar{\bar{x}}/e^{\alpha}$$

where

$$\bar{\bar{x}} = \left[\prod_{i=1}^{n} \bar{\bar{x}}_i\right]^{1/n} \qquad (4\text{-}17)$$

is the sample geometric mean, is distributed lognormally with parameters 0 and β^2/n. Also for a random sample size m from $f(y)$, it is known that

$$Q_2 = 2m\bar{y}/\mu_y$$

where

$$\bar{y} = \frac{1}{m}\sum_{i=1}^{m} y_i,\ \mu_y = E(y) \qquad (4\text{-}18)$$

is distributed as chi-square with 2m degrees of freedom. The quantities Q_1 and Q_2 are clearly independent. Consequently, if we let

$$w = Q_2/Q_1 = 2e^{\alpha}m\bar{y}/\mu_y\bar{\bar{x}} \qquad (4\text{-}19)$$

then it is known (Ref 35) that w has a probability density function g given by

$$g(w) = \begin{cases} \dfrac{(n/\beta^2)^{\frac{1}{2}}}{\Gamma(m)\ 2^m(2\pi)^{\frac{1}{2}}} \displaystyle\int_0^{\infty} \exp\left[-\frac{n}{2\beta^2}\ \ell n^2 z - \tfrac{1}{2}wz\right] x(wz)^{m-1}\partial z & 0 < w < \infty \\ 0 & \text{elsewhere} \end{cases} \qquad (4\text{-}20)$$

The variable w can now be used to obtain a lower confidence limit for the availability:

$$A = \frac{1}{1 + \exp[\alpha + \beta^2/2]/\mu_y}$$

as follows. For a given n/β^2 and m, we may obtain a number b_c such that

$$p[w < b_c] = p\left[\frac{2e^{\alpha}m\bar{y}}{\mu_y\bar{\bar{x}}} < b_c\right] = C$$

$$p\left[\frac{\exp[\alpha + \beta^2/2]}{\mu_y} < \frac{b_c\ \exp[\beta^2/2]}{2m}\left(\frac{\bar{\bar{x}}}{\bar{y}}\right)\right] = C$$

by algebraic manipulation

$$p\left[\frac{1}{1 + b_c\ \exp[\beta^2/2]\bar{\bar{x}}/2m\bar{y}} < A\right] = C \qquad (4\text{-}21)$$

Therefore, the 100C percent lower confidence limit (LCL) is given by

$$\mathrm{LCL} = \frac{2m\bar{y}}{2m\bar{y} + b_c \exp[\beta^2/2]\bar{\bar{x}}} \qquad (4\text{-}22)$$

where b_c is obtained from the tables of Gray and Lewis (Ref 35).

Gray and Schucany (Ref 36) gave some figures which show the lower confidence limits versus the statistic $\bar{\bar{x}}/\bar{y}$ for several values of m and n for the single value $\beta^2 = 1.0$. Also, they gave some figures for different β^2, but these figures are insensitive of the LCL to the number of observed failures m and repairs n that comprise the statistic. This insensitivity makes the figures difficult to read, since for any given problem, it is unlikely that m, n, and β^2 would match those figures.

V. Mont Carlo Comparisons Confidence Limits for Availability and Reliability

Summary

Reliability and maintainability engineering are recent and related engineering disciplines that make extensive use of mathematical techniques. In analyzing the dynamics of physical systems, certain probabilistic concepts have been developed in order to account for and explain random observations (i.e., failures or repairs). One of these concepts embodies the theory of Markov processes, the idea that the past has no effect on the future except through the present. The applicability of Markov process theory and techniques to the study of reliability and maintainability engineering has been shown in earlier studies.

A system is designed to achieve a given performance and its quality is the degree to which it meets this performance specification. Performance is normally specified in terms of the acceptable limits of such parameters as the maximum permissible noise or the minimum acceptable output power of a speech transmission channel, the maximum number of lost calls or the maximum switching time in a telephone exchange, the stability of a power supply or the frequency limits of an oscillator, and so on. It follows that system failure is defined as a departure from these specified limits.

Failure may be defined at many levels whereas only a system failure gives rise to complete loss of system use. A unit or sub-system failure may or may not give rise to a system failure depending upon the presence, or otherwise, of redundancy. Redundancy is a design configuration, as in the duplication or triplication of units within a control system, whereby the failure of some part of the system does not result in a system failure. Reliability is frequently enhanced by the use of redundancy, but the total number of unit failures requiring repair, hence the amount of maintenance, is usually increased due to the additional equipment. Since the availability is a measure of the ratio of the operating time of the system to the operating time plus the downtime, thus it includes both reliability and maintainability.

This thesis concentrates mainly on the confidence limits for the asymptotic availability of maintained systems. Confidence limits for the availability A(t) and reliability R(t) of maintained systems may be obtained in exactly the same method as applied to the two cases of steady state studies in this thesis as long as equations for the case have been derived (see Chapter III, equations 3-5, 3-7, 3-8). Using Table 3.2, a large number of systems can be modeled.

Results

Case 1. Exponential failure time

Exponential repair time

Number of repetitions = 500

Mean time between failure = 100 hours

Mean time to repair = 2 hours

$$\text{Exact availability} = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}}$$

$$= 100/100 + 2 = .98$$

Table 5.1 Results of the Double Mont Carlo Technique

Exponential Failure and Repair Times

Sample Size	No. of Trials per Repetition	Actual Percentage Coverage 95%	90%	85%	80%
10	100	100	91.6	85.8	80.2
20	100	100	89.6	85	79.2
30	100	100	89.8	85	80.2
10	200	95	89.2	84.2	79.4
20	200	95.6	92.4	87.8	82.2
30	200	95.6	90.8	87	82
10	500	94.4	90.4	85	80.4
20	500	94.8	90.6	84.8	78
30	500	95.4	90.8	86	82.2

Case 2. Exponential failure time

Lognormally repair time

Number of repetitions = 500

MTBF = 100 hours

MTTR = 2 hours

Exact availability = .98

Table 5.2 Results of the Double Mont Carlo Technique

Exponential Failure Time and Lognormally Repair Time

Sample Size	No. of Trials per Repetition	Actual Percentage of Coverage			
		95%	90%	85%	80%
10	100	100	92.1	91.3	87.4
20	100	100	88.9	82.3	81
30	100	100	91.4	86.2	80.5
10	200	96	92.3	88.5	82.3
20	200	99.3	92.2	86.1	82.8
30	200	99.3	93.6	88.1	81.4
10	500	99.3	93.8	87.4	83.6
20	500	99.3	93.6	86.7	82.8
30	500	99.3	92.8	88.1	81.5

We get accurate results in the case of exponential failure time and exponential repair time. A set of optimistic results was obtained in the case of the lognormal repair time with exact availability .98; that is, when repair times were correctly assumed to be lognormally distributed, the coverage was greater than in the situation of exponential repair time. The same conclusion has been obtained by Gray and Schycany (Ref 36). Better results can be obtained with smaller exact availability.

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APPENDICES

APPENDIX A

MAJOR REFERENCE SOURCES

Guides to Books

1. Card Catalog

Guides to Periodicals, Journals, Reports, etc.

1. Applied Science and Technology Index
2. Defense Documentation Center Bibliographic Search Facilities
3. The Engineering Index (Engineering Index, Inc.)
4. Guide to Library Research Resources (Air Force Institute of Technology Library, August 1969)
5. International Abstracts in Operations Research (Operations Research Society of America)
6. International Aerospace Abstracts (American Institute of Aeronautics and Astronautics, Inc.)
7. Mathematical Reviews (The American Mathematical Society)
8. National Aeronautics and Space Administration Literature Search Facilities
9. PANDEX Current Index to Scientific and Technical Literature (CCM Information Sciences, Inc.)
10. Quality Control and Applied Statistics (Executive Sciences Institute, Inc.)
11. Reliability Abstracts and Technical Reviews (National Aeronautics and Space Administration)
12. Scientific and Technical Aerospace Reports (National Aeronautics and Space Administration)
13. Selected RAND Abstracts (The RAND Corporation)
14. Statistical Theory and Method Abstracts (The International Statistical Institute)
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APPENDIX B

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This thesis presents the results of an extensive literature search for finding the confidence intervals for system reliability and availability of maintained systems using Mont Carlo techniques. The characteristics of system reliability and maintainability analysis are discussed. The basic Markovian failure and repair models are developed. A summary of the point estimates of availability, reliability of maintained

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systems, and the exact analytical methods are presented. Finally, the Bootstrap(Double Mont Carlo technique) is used to obtain the confidence limits for the availability of the maintained systems.